Properties of Triangles

Question1

In $\triangle ABC$, if $C = 120^\circ, c = \sqrt{19}$ and $b = 3$, then $a =$

AP EAPCET 2025 - 26th May Morning Shift

Options:

A.

4

B.

5

C.

2

D.

$\sqrt{5}$

Answer: C

Solution:

$b = 3, c = \sqrt{19}$ and $\angle C = 120^\circ$

$$\begin{aligned}
\because\quad & c^2 = a^2 + b^2 - 2ab\cos C \\
\Rightarrow\quad & 19 = a^2 + 9 - 6a \cdot \cos 120^\circ \\
\Rightarrow\quad & 19 = a^2 + 9 + 3a \\
\Rightarrow\quad & a^2 + 3a - 10 = 0 \\
\Rightarrow\quad & a^2 + 5a - 2a - 10 = 0 \\
\Rightarrow\quad & (a+5)(a-2) = 0 \\
\because\quad & a \neq -5
\end{aligned}$$

Hence, $a=2$

Question2

In a $\triangle ABC, 2A+C=300^{\circ}$. If the circumradius of the $\triangle ABC$ is eight times its inradius, then $\sin \frac{C}{2}=$

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Options:

A.

$\frac{1}{2}$

B.

$\frac{1}{4}$

C.

$\frac{3}{4+\sqrt{3}}$

D.

$\frac{1}{\sqrt{2}+1}$

Answer: B

Solution:

$\because 2A+C=300^{\circ} \quad \ldots(i)$

and $A+B+C=180^{\circ} \quad \ldots(ii)$

$\Rightarrow(2A+C)-(A+B+C)=300^{\circ}-180^{\circ}=120^{\circ}$
$\Rightarrow \quad A-B=120^{\circ} \Rightarrow A=B+120^{\circ}$

By Eq. (i)

$2B+C=60^{\circ} \Rightarrow C=60^{\circ}-2B$

given that $R=8r$

and $\frac{r}{R}=4\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$
$\Rightarrow \frac{1}{8}=4\sin\left(\frac{A}{2}\right)\cdot\sin\left(\frac{B}{2}\right)\cdot\sin\left(\frac{C}{2}\right)$

substitute values

$\frac{1}{8\times4}=\sin\left(\frac{B}{2}+60^\circ\right)\sin\left(\frac{B}{2}\right)\sin\left(30^\circ+B\right)$

let $B/2=u$

$$\frac{1}{32}=\left(\frac{1}{2}\sin u+\frac{\sqrt{3}}{2}\cos u\right)$$
$$\sin u\left(\frac{1}{2}\cos 2u-\frac{\sqrt{3}}{2}\sin 2u\right)$$
$$\Rightarrow \frac{1}{32}=\frac{1}{4}\sin u(\sin u+\sqrt{3}\cos u)$$
$$(\cos 2u-\sqrt{3}\sin 2u)$$

$$\Rightarrow \frac{1}{32}=\frac{1}{4}\sin u\cdot(2\sin(u+60^\circ)).2\cos(2u+60^\circ)$$

$$\Rightarrow \frac{1}{32}=\sin u\cdot\sin(u+60^\circ)\cdot\cos(2u+60^\circ)$$
$$\Rightarrow \frac{1}{32}=\left[\frac{1}{2}(\cos(u-u-60^\circ)-\cos(u+u+60^\circ)\right]\cdot\cos(2u+60^\circ)$$

$$\Rightarrow \frac{1}{32}=\left[\frac{1}{4}-\frac{1}{2}\cos(2u+60^\circ)\right]\cdot\cos(2u+60^\circ)$$
let $\cos(2u+60^\circ)=t$
$$\therefore \quad \frac{1}{32}=\left(\frac{1}{4}-\frac{1}{2}t\right)t$$
$$\Rightarrow \quad \frac{1}{32}=\frac{t}{4}-\frac{t^2}{2}$$
$$\Rightarrow \quad \frac{1}{8}=t-2t^2$$
$$\Rightarrow \quad 16t^2-8t+1=0$$
$$\therefore \quad t=\frac{1}{4}$$

$\Rightarrow \cos(2u+60^\circ)=\frac{1}{4}$

put $u=B/2$

$\Rightarrow \cos(B+60^\circ)=\frac{1}{4}$

Now, $C/2=30^\circ-B$

and

$\sin C/2=\sin(30^\circ-B)=\cos(90^\circ-(130^\circ-B))$

Hence, $\sin C/2=1/4$

Question3

In $\triangle ABC$, if $a=5, b=4$ and $\cos(A-B)=\frac{31}{32}$, then $c=$

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Options:

A.

8

B.

$\sqrt{41}$

C.

6

D.

$\sqrt{24}$

Answer: C

Solution:

$$\because \cos(A-B)=\cos A\cos B+\sin A\sin B$$

$$\text{and } \cos A=\frac{b^2+c^2-a^2}{2bc}$$

$$=\frac{16+c^2-25}{2\times4\times c}=\frac{c^2-9}{8c}$$

$$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{c^2+9}{10c}$$

$$\text{also, } \sin A=\sqrt{1-\cos^2 A} \text{ and}$$

$$\sin B=\sqrt{1-\cos^2 B}$$

$$\Rightarrow \frac{31}{32}=\left(\frac{c^2-9}{8c}\right)\left(\frac{c^2+9}{10c}\right)+\left(\sqrt{1-\left(\frac{c^2-9}{8c}\right)^2}\right)\left(\sqrt{1-\left(\frac{c^2+9}{10c}\right)^2}\right)$$

from the options

put $C=6$

$\cos A=\dfrac{36-9}{8\times 6}=9/16$ and

$\cos B=\dfrac{36+9}{10\times 6}=3/4$

$\sin A=\sqrt{1-\left(\dfrac{9}{16}\right)^2}=\dfrac{\sqrt{175}}{16}$

$\sin B=\sqrt{1-(3/4)^2}=\sqrt{7}/4$

$\therefore \dfrac{9}{16}\times\dfrac{3}{4}+\dfrac{\sqrt{175}}{16}\times\dfrac{\sqrt{7}}{4}$

$=\dfrac{27}{64}+\dfrac{35}{64}=31/32$

Hence, $c=6$ satisfy the equation

Question4

In a $\triangle ABC$, if $A=30^\circ$ and $\frac{b}{(\sqrt{3}+1)^2+2(\sqrt{2}-1)}=\frac{c}{(\sqrt{3}+1)^2-2(\sqrt{2}-1)}$, then B

AP EAPCET 2025 - 26th May Evening Shift

Options:

A.

60°

B.

97.5°

C.

75°

D.

52.5°

Answer: B

Solution:

Given,

$$A = 30^\circ$$

$$\frac{b}{(\sqrt{3}+1)^2 + 2(\sqrt{2}-1)} = \frac{c}{(\sqrt{3}+1)^2 - 2(\sqrt{2}-1)}$$

$$\Rightarrow \frac{b}{c} = \frac{(\sqrt{3}+1)^2 + 2(\sqrt{2}-1)}{(\sqrt{3}+1)^2 - 2(\sqrt{2}-1)}$$

By compounds and dividendo

$$\frac{b+c}{b-c} = \frac{2(\sqrt{3}+1)^2}{4(\sqrt{2}-1)} \Rightarrow \frac{b+c}{b-c} = \frac{2+\sqrt{3}}{\sqrt{2}-1}$$

By compounds and dividendo

$$\frac{2b}{2c} = \frac{2+\sqrt{3}+\sqrt{2}-1}{2+\sqrt{3}-\sqrt{2}+1}$$

$$\Rightarrow \frac{b}{c} = \frac{\sqrt{3}+\sqrt{2}+1}{\sqrt{3}-\sqrt{2}+3} \Rightarrow \frac{\sin B}{\sin C} = \frac{\sqrt{3}+\sqrt{2}+1}{\sqrt{3}-\sqrt{2}+3}$$

$$\Rightarrow \frac{\sin C}{\sin B} = \frac{3+\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}+1}$$

$$\Rightarrow \frac{\sin(150^\circ - B)}{\sin B} = \frac{3+\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}+1}$$

$$\{\because A = 30^\circ \Rightarrow B + C = 150^\circ\}$$

$$\Rightarrow \frac{1}{2}\cdot\frac{\cos B}{\sin B} + \frac{\sqrt{3}\sin B}{2}\frac{3+\sqrt{3}-\sqrt{2}}{\sin B} = \frac{\sqrt{3}+\sqrt{2}+1}{2}$$

$$\Rightarrow \frac{\cot B}{2} = \frac{3+\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}+1} - \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{\cot B}{2} = \frac{6+2\sqrt{3}-2\sqrt{2}-3-\sqrt{6}-\sqrt{3}}{2(\sqrt{3}+\sqrt{2}+1)}$$

$$\cot B = \frac{3+\sqrt{3}-2\sqrt{2}-\sqrt{6}}{\sqrt{3}+\sqrt{2}+1} < 0$$

$\because \cot B =$ Negative

$\therefore \quad B > 90^\circ$

Question5

In $\triangle ABC$ is the line joining the circumcentre and the incentre is parallel to BC, then $\cos B + \cos C =$

AP EAPCET 2025 - 26th May Evening Shift

Options:

A.

$1/2$

B.

$3/4$

C.

1

D.

$3/2$

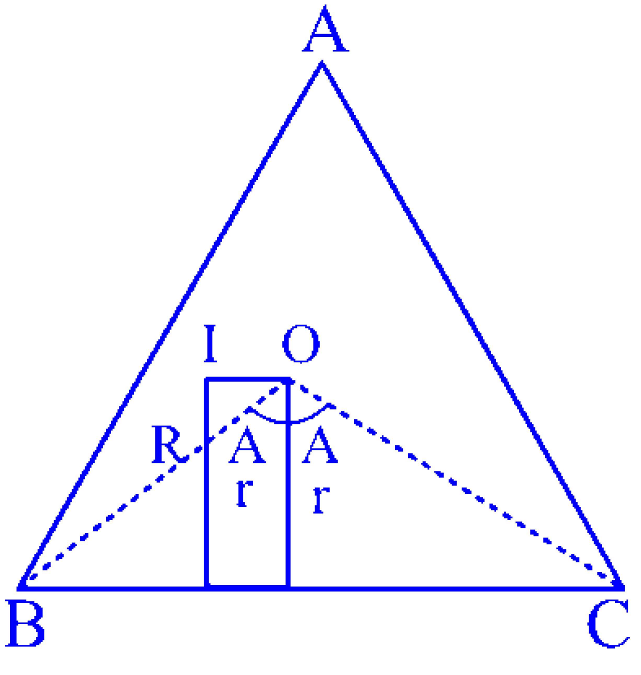
Answer: C

Solution:

O-circumcentre

$I =$ Incentre

$\cos A = \dfrac{r}{R}$



$\because \cos A + \cos B + \cos C$

$= 1 + 4 \sin \dfrac{A}{2} \sin \dfrac{B}{2} \sin \dfrac{C}{2}$

$$= 1 + 4 \cdot \frac{r}{4R}\left(\because r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$$

$$= 1 + \frac{r}{R}$$

$$\therefore \frac{r}{R} + \cos B + \cos C = 1 + \frac{r}{R}$$

$$\Rightarrow \cos B + \cos C = 1$$

Question6

In a $\triangle ABC$, if $r_1 : r_2 = 3 : 4$ and $r_2 : r_3 = 2 : 3$, then a :b :c=

AP EAPCET 2025 - 26th May Evening Shift

Options:

A.

$2 : 3 : 4$

B.

$3 : 4 : 5$

C.

$4 : 5 : 6$

D.

$5 : 6 : 7$

Answer: D

Solution:

Let's first find the common ratio $r_1 : r_2 : r_3$.

We are given:

1. $r_1 : r_2 = 3 : 4$
2. $r_2 : r_3 = 2 : 3$

To combine these ratios, we need to make the value corresponding to r_2 common.

We can write the second ratio as:

$r_2 : r_3 = 2 : 3 = (2 \times 2) : (3 \times 2) = 4 : 6.$

Now, we have:

$r_1 : r_2 = 3 : 4$

$r_2 : r_3 = 4 : 6$

Combining them gives the ratio $r_1 : r_2 : r_3 = 3 : 4 : 6$.

Next, we use the formulas for the exradii r_1, r_2, r_3 in terms of the area of the triangle Δ and the semi-perimeter $s = \frac{a+b+c}{2}$:

$r_1 = \frac{\Delta}{s-a}$

$r_2 = \frac{\Delta}{s-b}$

$r_3 = \frac{\Delta}{s-c}$

So, the ratio $r_1 : r_2 : r_3$ can be written as:

$\frac{\Delta}{s-a} : \frac{\Delta}{s-b} : \frac{\Delta}{s-c} = 3 : 4 : 6.$

Dividing by Δ (since $\Delta \neq 0$ for a triangle), we get:

$\frac{1}{s-a} : \frac{1}{s-b} : \frac{1}{s-c} = 3 : 4 : 6.$

This implies that there exists some constant k (let's use K to avoid confusion with k for side ratios) such that:

$\frac{1}{s-a} = 3K \implies s - a = \frac{1}{3K}$

$\frac{1}{s-b} = 4K \implies s - b = \frac{1}{4K}$

$\frac{1}{s-c} = 6K \implies s - c = \frac{1}{6K}$

Let's denote $1/K$ as C for simplicity. Then:

$s - a = C/3$

$s - b = C/4$

$s - c = C/6$

We know the relationships between the sides a, b, c and $s - a, s - b, s - c$:

$a = (s - b) + (s - c)$

$b = (s - a) + (s - c)$

$c = (s - a) + (s - b)$

Substituting the expressions for $s - a, s - b, s - c$:

$a = \frac{C}{4} + \frac{C}{6} = \frac{3C+2C}{12} = \frac{5C}{12}$

$b = \frac{C}{3} + \frac{C}{6} = \frac{2C+C}{6} = \frac{3C}{6} = \frac{C}{2}$

$c = \frac{C}{3} + \frac{C}{4} = \frac{4C+3C}{12} = \frac{7C}{12}$

Now we can find the ratio $a : b : c$:

$a : b : c = \frac{5C}{12} : \frac{C}{2} : \frac{7C}{12}$

To simplify the ratio, we can multiply all parts by $12/C$:

$a : b : c = \left(\frac{5C}{12} \times \frac{12}{C}\right) : \left(\frac{C}{2} \times \frac{12}{C}\right) : \left(\frac{7C}{12} \times \frac{12}{C}\right)$

$a : b : c = 5 : 6 : 7$

The final answer is $\boxed{5 : 6 : 7}$.

Question7

In a $\triangle ABC$, if a, b, c are in arithmetic progression and the angle A is twice the angle C, then $\cos A : \cos B : \cos C =$

AP EAPCET 2025 - 24th May Morning Shift

Options:

A.

$2 : 3 : 4$

B.

$3 : 4 : 8$

C.

$2 : 9 : 12$

D.

$1 : 9 : 6$

Answer: C

Solution:

Since, a, b and c are in AP

So, $2b = a + c$

Also, $\angle A = 2\angle C$ $\quad \ldots (i)$

Now, in a $\triangle ABC$,

$a = 2R\sin A, b = 2R\sin B, c = 2R\sin C$

So, $2b = a + c$

$\Rightarrow \quad 2 \cdot 2R\sin B = 2R\sin A + 2R\sin C$

$\Rightarrow \quad 2\sin B = \sin A + \sin C$

In a $\triangle ABC$,

$$A + B + C = 180^\circ$$
$$\Rightarrow 2C + B + C = 180^\circ$$
$$\Rightarrow B = 180^\circ - 3C$$

$$So, \sin B = \sin(180^\circ - 3C)$$
$$= \sin(3C) \quad \ldots (ii)$$

And $2\sin B = \sin A + \sin C$

$\Rightarrow 2\sin(3C) = \sin(2C) + \sin C$

[using Eq. (i) and (ii)]

$= 2\sin C\cos C + \sin C$

and $\sin(3C) = 3\sin C - 4\sin^3 C$

$$So, 2\sin(3C) = 2\left(3 - 4\sin^2 C\right)$$
$$= 2\cos C + 1$$
$$= 2\left(4\cos^2 C - 1\right)$$

$$\Rightarrow 8\cos^2 C - 2 = 2\cos C + 1$$
$$\Rightarrow 8\cos^2 C - 2\cos C - 3 = 0$$

$$\therefore \cos C = \frac{2 \pm \sqrt{4 - 4(8)(-3)}}{16}$$
$$= \frac{2 \pm \sqrt{100}}{16} = \frac{2 \pm 10}{16}$$

So, $\quad \cos C = \frac{3}{4}, \cos C = -\frac{1}{2}$

Since, C must be positive,

So, $\cos C = \frac{3}{4}$

Now, $\cos A = \cos(2C)$

$= 2\cos^2 C - 1 = 2\left(\frac{3}{4}\right)^2 - 1 = \frac{1}{8}$

And $B = 180^\circ - 3C$

$$\cos B = \cos(180^\circ - 3C)$$
$$= -\cos(3C)$$
$$= -\left(4\cos^3 C - 3\cos C\right)$$

$$= -\left(4\left(\frac{27}{64}\right) - 3 \cdot \frac{3}{4}\right)$$
$$= -\left(\frac{27}{16} - \frac{9}{4}\right)$$
$$= -\left(\frac{27-36}{16}\right) = \frac{9}{16}$$

$\therefore \cos B = \frac{9}{16}$

So, $\cos A : \cos B : \cos C$

$$= \frac{1}{8} : \frac{9}{16} : \frac{3}{4}$$
$$= \frac{2:9:12}{16}$$

So, $\cos A : \cos B : \cos C = 2 : 9 : 12$

Question8

In a $\triangle ABC$, A, B and C are in arithmetic progression, $rr_3 = r_1 r_2$ and $c = 10$, then $a^2 + b^2 + c^2 =$

AP EAPCET 2025 - 24th May Morning Shift

Options:

A.

128

B.

392

C.

288

D.

200

Answer: D

Solution:

In a triangle A, B and C are in arithmetic progression.

So, $B=\frac{\pi}{3}$

Now, $r r_3=r_1 r_2$

$$\begin{aligned}
& \Rightarrow \quad \frac{\Delta}{s} \cdot \frac{\Delta}{s-c}=\frac{\Delta}{s-a} \cdot \frac{\Delta}{s-b} \\
& \Rightarrow \quad \frac{1}{s(s-c)}=\frac{1}{(s-a)(s-b)} \\
& \Rightarrow \quad s^2-(a+b) s+a b=s^2-c s \\
& \Rightarrow \quad a b=(a+b-c) s \\
& \Rightarrow \quad a b=(a+b-c) \frac{(a+b+c)}{2} \\
& \Rightarrow \quad 2 a b=(a+b)^2-c^2 \\
& \Rightarrow \quad 2 a b=a^2+b^2+2 a b-c^2 \\
& \Rightarrow \quad a^2+b^2=c^2 \quad \quad \ldots(i)
\end{aligned}$$

Also, using cosine rule,

$$\begin{aligned}
& b^2=a^2+c^2-2 a c \cos B \\
& \Rightarrow b^2=a^2+c^2-2 a c \cos \left(\frac{\pi}{3}\right) \\
& \Rightarrow b^2=a^2+c^2-a c \\
& \Rightarrow c^2-a^2=a^2+c^2-a c \\
& \Rightarrow 2 a^2=a c \Rightarrow 2 a=c \\
& \Rightarrow a=\frac{c}{2}=\frac{10}{2}=5
\end{aligned}$$

So, $b^2=c^2-a^2=100-25=75$

$\Rightarrow \quad b=5 \sqrt{3}$

Now, $a^2+b^2+c^2=25+75+100=200$

Question9

In a $\triangle A B C, \frac{2\left(r_1+r_3\right)}{a c(1+\cos B)}=$

AP EAPCET 2025 - 24th May Morning Shift

Options:

A.

$\frac{\Delta}{b}$

B.

$\frac{b}{\Delta}$

C.

$\frac{2\Delta}{2+b+c}$

D.

$\frac{a+b+c}{2\Delta}$

Answer: B

Solution:

$\text{LHS} = \frac{2(r_1+r_3)}{ac(1+\cos B)}$

$$= \frac{2\left(r_1+r_3\right)}{ac\left(1+2\cos^2\frac{B}{2}-1\right)}$$
$$= \frac{2\left(r_1+r_3\right)}{ac\left(2\cos^2\frac{B}{2}\right)}$$

We know that, $r_1 = \frac{\Delta}{s-a}, r_3 = \frac{\Delta}{s-c}$

And $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$

Hence, $\text{LHS} = \frac{2\left[\frac{\Delta}{s-a}+\frac{\Delta}{s-c}\right]}{ac\cdot 2\left(\sqrt{\frac{s(s-b)}{ac}}\right)^2}$

$$= \frac{2\Delta\left[\frac{1}{s-a}+\frac{1}{s-c}\right]}{2ac\cdot\left(\frac{s(s-b)}{ac}\right)}$$
$$= \frac{\Delta\left[\frac{s-c+s-a}{(s-a)(s-c)}\right]}{s(s-b)}$$
$$= \frac{\Delta(2s-a-c)}{s(s-a)(s-b)(s-c)}$$

$$= \frac{\Delta(a+b+c-a-c)}{s(s-a)(s-b)(s-c)}$$
$$= \frac{b\Delta}{s(s-a)(s-b)(s-c)}$$
$$= \frac{b}{\Delta} = \text{RHS}$$

Question10

In $\triangle ABC$, if $a = 8, b = 10, c = 12$, then $\frac{r}{R} =$

AP EAPCET 2025 - 23rd May Evening Shift

Options:

A.

$\frac{8}{15}$

B.

$\frac{7}{16}$

C.

$\frac{3}{5}$

D.

$\frac{5}{8}$

Answer: B

Solution:

$\because a = 8, b = 10$ and $c = 12$

and $r = \frac{\Delta}{s}, R = \frac{abc}{4\Delta}$

So, $\frac{r}{R} = \frac{4\Delta^2}{s \cdot abc}$

$\because \quad s = \frac{a+b+c}{2} = 15$

also, $\Delta = \sqrt{15(15-8)(15-10)(15-12)}$

$= \sqrt{15 \times 7 \times 5 \times 3} = 15\sqrt{7}$

Thus, $\frac{r}{R} = \frac{4 \times 225 \times 7}{15 \times 8 \times 10 \times 12} = \frac{7}{16}$

Question11

In $\triangle ABC$, if $a = 13, b = 8, c = 7$, then $\cos(B + C) =$

AP EAPCET 2025 - 23rd May Evening Shift

Options:

A. $\frac{11}{13}$

B. $\frac{23}{26}$

C. $\frac{3}{4}$

D. $\frac{1}{2}$

Answer: D

Solution:

In $\triangle ABC, \angle A + \angle B + \angle C = 180^\circ$

So, $\angle B + \angle C = 180^\circ - \angle A$

$$\begin{aligned}\Rightarrow \cos(B+C) &= \cos\left(180^\circ - A\right)\\ &= -\cos A\end{aligned}$$

$$\begin{aligned}Now,\ \cos A &= \frac{b^2+c^2-a^2}{2bc} = \frac{8^2+7^2-13^2}{2\times 8\times 7}\\ &= \frac{64+49-169}{112} = -\frac{56}{112}\\ &= -\frac{1}{2}\end{aligned}$$

Hence, $\cos(B+C) = -\left(-\frac{1}{2}\right) = \frac{1}{2}$

Question12

In a $\triangle ABC$, if $(r_1 - r_3)(r_1 - r_2) - 2r_2r_3 = 0$, then $a^2 - b^2 =$

AP EAPCET 2025 - 23rd May Evening Shift

Options:

A.

$c^2+\frac{b^2}{4}$

B.

c^2

C.

abc

D.

$\frac{b+a}{c}$

Answer: B

Solution:

$\because\left(r_1-r_3\right)\left(r_1-r_2\right)-2 r_2 r_3=0$

and we know that

$r_1=\frac{\Delta}{s-a}, r_2=\frac{\Delta}{s-b}, r_3=\frac{\Delta}{s-c}$

substitute these values

$$\Rightarrow \quad \Delta^2\left(\frac{1}{s-a}-\frac{1}{s-c}\right)\left(\frac{1}{s-a}-\frac{1}{s-b}\right)-2 \Delta^2\left(\frac{1}{(s-b)(s-c)}\right)=0$$

$$\Rightarrow \quad\left(\frac{s-c-s+a}{(s-a)(s-c)}\right)\left(\frac{s-b-s+a}{(s-a)(s-b)}\right)-\left(\frac{2}{(s-b)(s-c)}\right)=0$$

$$\begin{aligned}
& \Rightarrow \quad \frac{1}{(s-b)(s-c)}\left[\frac{(a-c)(a-b)}{(s-a)^2}-2\right]=0 \\
& \Rightarrow \quad(a-c)(a-b)-2(s-a)^2=0 \\
& \Rightarrow \quad(a-c)(a-b)-2\left(\frac{a+b+c}{2}-a\right)^2=0 \\
& \Rightarrow \quad 2(a-c)(a-b)-(b+c-a)^2=0 \\
& \Rightarrow \quad 2\left(a^2-ab-ac+bc\right)-\left(b^2+c^2+a^2\right)-2(bc-ca-ab)=0 \\
& \Rightarrow \quad a^2-b^2-c^2-2ab-2ac+2bc-2bc+2ac+2ab=0 \\
& \Rightarrow \quad a^2-b^2=c^2
\end{aligned}$$

Question13

If the median AD of the $\triangle ABC$ is bisected at E and BE meets AC in E, then $AF : AC=$

Options:

A.

1 : 4

B.

1 : 3

C.

1 : 2

D.

3 : 4

Answer: B

Solution:

$\because D$ is the mid-point of BC and E is the mid-point of AD.

Let BE and AC intersects at F

So, $\frac{AF}{FC} = \frac{1}{2}$

and $AC = AF + FC = \frac{1}{3}AC + \frac{2}{3}AC$

Thus, $\frac{AF}{AC} = \frac{\frac{1}{3}AC}{AC} = \frac{1}{3}$

Hence, $AF : AC = 1 : 3$

Question14

In $\triangle ABC$ if $\cos A \cos B + \sin A \sin B \sin C = 1$, then $\sin A + \sin B + \sin C =$

Options:

A.

$\frac{2+\sqrt{3}}{2}$

B.

$1+\sqrt{2}$

C.

$\frac{2\sqrt{3}-1}{2}$

D.

$\frac{3+\sqrt{3}}{2}$

Answer: B

Solution:

We have,

$\cos A \cos B+\sin A \cdot \sin B \sin C=1$

In $\triangle ABC$,

$\cos A \cos B+\sin A \sin B \sin C=1$ is possible if $A=45^{\circ}, B=45^{\circ}$ and $C=90^{\circ}$ i.e.

$$\begin{aligned}
&\cos 45^{\circ} \cdot \cos 45^{\circ}+\sin 45^{\circ} \cdot \sin 45 \cdot \sin 90^{\circ} \\
&=\frac{1}{2}+\frac{1}{2}=1 \\
&\therefore \sin A+\sin B+\sin C \\
&\quad=\sin 45^{\circ}+\sin 45^{\circ}+\sin 90^{\circ} \\
&\quad=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+1=\sqrt{2}+1
\end{aligned}$$

Question15

In $\triangle ABC$, if $a: b: c=4: 5: 6$, then $\frac{\cos A+3 \cos C}{\cos B}=$

AP EAPCET 2025 - 23rd May Morning Shift

Options:

A.

1

B.

4

C.

2

D.

3

Answer: C

Solution:

Given, $a : b : c = 4 : 5 : 6$

We can express the sides as $a = 4x$, $b = 5x$ and $c = 6x$ for some positive constant x.

$$\because \cos A = \frac{b^2 + c^2 - a^2}{2bc} (\text{ By Cosine rule})$$
$$= \frac{(5x)^2 + (6x)^2 - (4x)^2}{2 \times 5x \times 6x} = \frac{45x^2}{60x^2} = \frac{3}{4}$$

$$\text{Similarly, } \cos B = \frac{(4x)^2 + (6x)^2 - (5x)^2}{2(4x)(6x)}$$
$$= \frac{27x^2}{48x^2} = \frac{9}{16}$$
$$\text{and } \cos C = \frac{(4x)^2 + (5x)^2 - (6x)^2}{2(4x)(5x)}$$
$$= \frac{5x^2}{40x^2} = \frac{1}{8}$$

$$\text{So, } \frac{\cos A + 3\cos C}{\cos B} = \frac{\frac{3}{4} + 3\left(\frac{1}{8}\right)}{\frac{9}{16}} = \frac{\frac{6+3}{8}}{\frac{9}{16}}$$
$$= \frac{9}{8} \times \frac{16}{9} = 2$$

Question16

In $\triangle ABC$, if $a = 6, b = 8$ and $c = 10$, then $\frac{2r_2r_3}{rr_1} =$

Options:

A.

$b+c$

B.

$c+a$

C.

$a+b$

D.

$a+b+c$

Answer: A

Solution:

Given, $a=6, b=8, c=10$

$$\therefore \quad s=\frac{a+b+c}{2}=12$$

$$\text{and area } (\Delta)=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{12 \times 6 \times 4 \times 2}=24$$

Now, $r_1=\frac{\Delta}{s-a}=\frac{24}{6}=4$ and

$$r=\frac{\Delta}{s}=\frac{24}{12}=2$$
$$r_2=\frac{\Delta}{s-b}=\frac{24}{4}=6$$
$$r_3=\frac{\Delta}{s-c}=\frac{24}{2}=12$$
$$\therefore \quad \frac{2 r_2 r_3}{r_1}=\frac{2 \times 6 \times 12}{2 \times 4}=18$$
$$=8+10=b+c$$

Question17

If the sides a, b, c of the $\triangle ABC$ are in harmonic progression, then $\operatorname{cosec}^2 A/2, \operatorname{cosec}^2 B/2, \operatorname{cosec}^2 C/2$ are in

Options:

A.

Arithmetico-geometric progression

B.

Arithmetic progression

C.

Geometric progression

D.

Harmonic progression

Answer: B

Solution:

a, b, c are in HP.

$\Rightarrow b = \frac{2ac}{a+c}$

Let $\operatorname{cosec}^2 \frac{A}{2}, \operatorname{cosec}^2 \frac{B}{2}, \operatorname{cosec}^2 \frac{C}{2}$ are in AP .

$$\Rightarrow \frac{1}{\sin^2 \frac{A}{2}}, \frac{1}{\sin^2 \frac{B}{2}}, \frac{1}{\sin^2 \frac{C}{2}} \text{ are in AP.}$$
$$\Rightarrow \frac{bc}{(s-b)(s-c)}, \frac{ac}{(s-a)(s-c)}$$
$$\frac{ab}{(s-a)(s-b)} \text{ are in AP.}$$
$$\Rightarrow \frac{bc}{s(s-b)(s-c)}, \frac{ac}{s(s-a)(s-c)}$$
$$\frac{ab}{s(s-a)(s-b)} \text{ are in AP.}$$

$\Rightarrow \frac{bc(s-a)}{s(s-a)(s-b)(s-c)}, \frac{ac(s-b)}{s(s-a)(s-b)(s-c)'} \frac{ab(s-c)}{s(s-a)(s-b)(s-c)}$ are in AP.

$\Rightarrow bcs - abc, acs - abc, abs - abc$ are in AP.

$\Rightarrow bcs, acs, abs$ are in AP.

$\Rightarrow bc, ac, ab$ are in AP.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP.

$\Rightarrow a, b, c$ are in HP. Which is true.

Question18

In $\triangle ABC$, if $r = 3$ and $R = 5$, then $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} =$

AP EAPCET 2025 - 22nd May Evening Shift

Options:

A.

$\frac{1}{30}$

B.

$\frac{12}{15}$

C.

$\frac{1}{15}$

D.

$\frac{5}{36}$

Answer: A

Solution:

$$\begin{aligned} & r = 3, R = 5 \\ & \frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = \frac{a+b+c}{abc} = \frac{2s}{abc} \\ & \qquad = \frac{2s}{abc} \times \frac{\Delta}{\Delta} \\ & \qquad = \frac{1}{2} \frac{4\Delta}{abc} \cdot \frac{s}{\Delta} \\ & \qquad = \frac{1}{2} \cdot \frac{1}{R} \cdot \frac{1}{r} = \frac{1}{2} \cdot \frac{1}{5} \cdot \frac{1}{3} = \frac{1}{30} \end{aligned}$$

Question19

In a $\triangle ABC, A-B=120^{\circ}, R=8r$, then $\frac{1+\cos C}{1-\cos C}=$

AP EAPCET 2025 - 22nd May Morning Shift

Options:

A.

16

B.

14

C.

15

D.

10

Answer: C

Solution:

Given in $\triangle ABC$,

$$A-B=120^{\circ}, R=8r$$

$$\frac{1+\cos C}{1-\cos C}$$

$$\Rightarrow R=8\left(4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$$

$$\Rightarrow 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\frac{1}{16}$$

$$\Rightarrow\left(\cos\frac{A-B}{2}-\cos\left(\frac{A+B}{2}\right)\right)\sin\frac{C}{2}=\frac{1}{16}$$

$$\Rightarrow \sin\frac{C}{2}\left(\frac{1}{2}-\sin\frac{C}{2}\right)=\frac{1}{16}$$

$$\sin\frac{C}{2}=\frac{1}{4}$$

$$\cos C=1-\frac{2}{16}=\frac{7}{8}$$

$$\Rightarrow \frac{1+\cos C}{1-\cos C}=15$$

Question20

In $\triangle ABC, \sqrt{\frac{r \cdot r_2}{r_3 r_1}}=$

AP EAPCET 2025 - 22nd May Morning Shift

Options:

A.

$\left(r_3-r_2\right)\left(r_1-r_2\right)$

B.

r_3+r_1

C.

$\frac{b}{r_3-r_1}$

D.

$\frac{b}{r_3+r_1}$

Answer: D

Solution:

Given, $\sqrt{\frac{r_2}{r_3 r_1}}$, in $\triangle ABC$

$$\sqrt{\frac{r_2}{r_3 r_1}} = \sqrt{\frac{\frac{\Delta}{s} - \frac{\Delta}{s-b}}{\frac{\Delta}{s-c}\frac{\Delta}{s-a}}}\sqrt{\frac{\frac{\Delta}{s}\frac{\Delta}{s-b}}{\frac{\Delta}{s-c}\frac{\Delta}{s-a}}}$$

$$= \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\Rightarrow s = \frac{a+b+c}{2}$$

$$\Rightarrow 2s = a+b+c \Rightarrow 2s-a-c = b$$

$$Now, \frac{b}{r_3 + r_1} = \frac{b}{\frac{\Delta}{s-c} + \frac{\Delta}{s-a}}$$

$$= \frac{b}{\frac{\Delta(2s-a-c)}{(s-a)(s-c)}} = \frac{b(s-a)(s-c)}{\Delta b}$$

$$= \frac{(s-a)(s-c)}{\sqrt{s(s-a)(s-b)(s-c)}}$$

$$= \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} = \sqrt{\frac{rr_2}{r_3 r_1}}$$

Question21

If $A(0,0,0) B(3,4,0)$ and $C(0,12,5)$ are the vertices of a $\triangle ABC$, then the x-coordinate of its incentre is

AP EAPCET 2025 - 22nd May Morning Shift

Options:

A.

$\frac{25}{18+7\sqrt{2}}$

B.

$\frac{25}{26}$

C.

$\frac{39}{18+7\sqrt{2}}$

D.

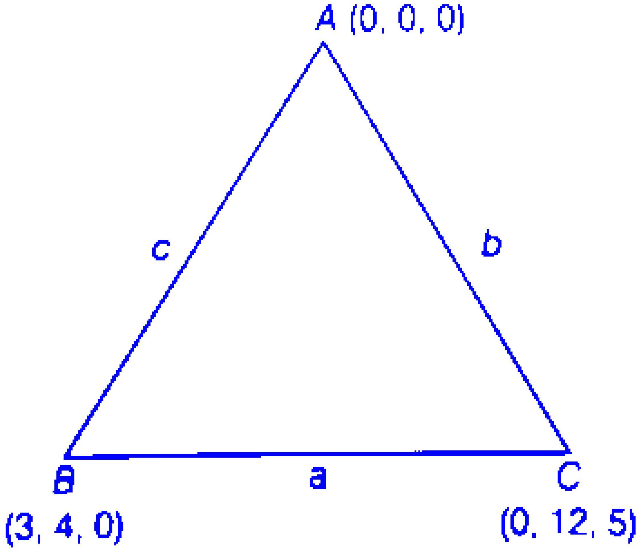
$\frac{39}{26}$

Answer: C

Solution:

x-coordinate of incentre

$$= \frac{ax_1 + by_1 + cz_1}{a+b+c}$$



$a = \sqrt{98}$

$b = 13$

$c = 5$

$$I_x = \frac{\sqrt{98} \times 0 + 13 \times 3 + 5 \times 0}{18 + \sqrt{98}}$$

$$= \frac{39}{18 + \sqrt{98}} = \frac{39}{18 + 7\sqrt{2}}$$

Question22

In a $\triangle ABC$, if $\sin \frac{A}{2} = \frac{1}{4}\sqrt{\frac{3}{5}}$, $a = 2$, $c = 5$ and b is an integer, then the area (in sq. units) of $\triangle ABC$ is

AP EAPCET 2025 - 21st May Evening Shift

Options:

A.

$\frac{\sqrt{297}}{4}$

B.

$\frac{\sqrt{231}}{4}$

C.

$\frac{\sqrt{385}}{4}$

D.

$\frac{\sqrt{185}}{4}$

Answer: B

Solution:

$\sin \frac{A}{2} = \frac{1}{4}\sqrt{\frac{3}{5}} a = 2c = 5$

$$\cos A = 1 - 2\sin^2 \frac{A}{2} = 1 - 2 \cdot \frac{1}{16} \times \frac{3}{5} = \frac{37}{40}$$

$$\text{Now, } \cos A = \frac{b^2 + c^2 - a^2}{2 \times b \times c}$$

$$\Rightarrow \quad \frac{37}{40} = \frac{b^2 + 25 - 4}{2 \times b \times 5}$$

$$\Rightarrow \quad \frac{37}{4} = \frac{b^2 + 21}{b}$$

$$\Rightarrow 4b^2 - 37b + 84 = 0$$

$$\therefore (4b - 21)(b - 4) = 0$$

$\therefore b = 4$ because given b is an integer

$$\because \sin A=\sqrt{1-\cos^2 A}=\sqrt{1-\left(\frac{37}{40}\right)^2}$$

$$\therefore \text{ Area of triangle }=\frac{1}{2} b \times c \times \sin A$$

$$=\frac{1}{2} \times 4 \times 5 \sqrt{1-\left(\frac{37}{40}\right)^2}$$

$$=10 \sqrt{\frac{77}{40} \times \frac{3}{40}}=\frac{1}{4} \sqrt{231}$$

Question23

In a $\triangle ABC$ if $a+c=5b$, then $\cot \frac{A}{2} \cot \frac{C}{2}=$

AP EAPCET 2025 - 21st May Evening Shift

Options:

A.

2

B.

$\frac{1}{2}$

C.

$\frac{3}{2}$

D.

$\frac{2}{3}$

Answer: C

Solution:

Given, $a+c=5b$
$a+b+c=2s=6b$
$s=3b$
$\cot\frac{A}{2}\cdot\cot\frac{C}{2}=\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$
$\times\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$
$$=\frac{s}{s-b}=\frac{3b}{3b-b}=\frac{3b}{2b}=\frac{3}{2}$$

Question24

In a $\triangle ABC$, if $r_1=3, r_2=4, r_3=6$, then $b=$

AP EAPCET 2025 - 21st May Evening Shift

Options:

A.

$2\sqrt{6}$

B.

$\frac{5\sqrt{6}}{3}$

C.

$\frac{7\sqrt{6}}{3}$

D.

$3\sqrt{6}$

Answer: A

Solution:

Given, $r_1=3, r_2=4$ and $r_3=6$

$$r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$$
$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{s}{\Delta}$$
$$\frac{1}{3} + \frac{1}{4} + \frac{1}{6} = \frac{s}{\Delta} \Rightarrow \frac{s}{\Delta} = \frac{3}{4}$$

And $r_1 r_2 + r_2 r_3 + r_1 r_3 = s^2$

$\Rightarrow \quad 3 \times 4 + 4 \times 6 + 3 \times 6 = s^2$

$\Rightarrow \quad 12 + 24 + 18 = s^2$

$\Rightarrow \quad s^2 = 54$

$\Rightarrow \quad s = 3\sqrt{6}$

$\therefore \quad \Delta = \frac{4s}{3} = \frac{12\sqrt{6}}{3} = 4\sqrt{6}$

$r_2 = \frac{\Delta}{s-b} \Rightarrow s - b = \frac{\Delta}{r_2}$

$b = s - \frac{\Delta}{r_2} = 3\sqrt{6} - \frac{4\sqrt{6}}{4}$

$= 3\sqrt{6} - \sqrt{6} = 2\sqrt{6}$

Question25

In $\triangle ABC$, the sum of the lengths of two sides is x and the product of those lengths is y. If c is the length of its third side and $x^2 - c^2 = y$, then the circumradius of that triangle is

AP EAPCET 2025 - 21st May Morning Shift

Options:

A.

$\frac{c}{\sqrt{3}}$

B.

$\frac{c}{3}$

C.

$\frac{y}{\sqrt{3}}$

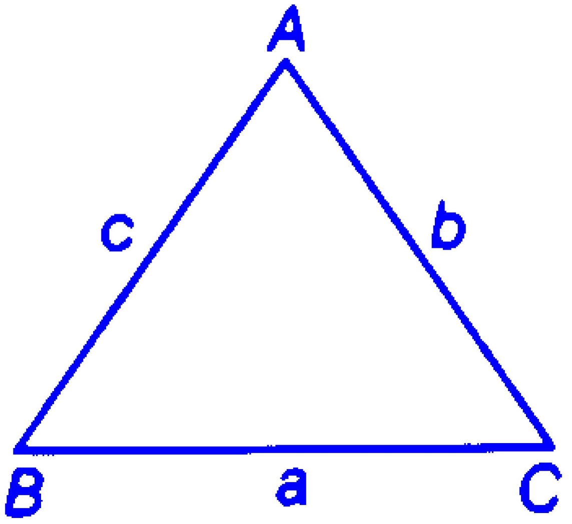
D.

$\frac{3y}{2}$

Answer: A

Solution:

We have,
$x=a+b$
$y=ab$



We have, $x^2-c^2=y$

$$\Rightarrow \quad (a+b)^2-c^2=ab$$
$$\Rightarrow \quad a^2+b^2-c^2=-ab$$
$$\Rightarrow \quad \frac{a^2+b^2-c^2}{2ab}=-\frac{1}{2}$$
$$\cos c=-\frac{1}{2} \Rightarrow c=\frac{2\pi}{3} \Rightarrow \sin c=\frac{\sqrt{3}}{2}$$

Using circumradius formula,

$c=2R\sin c$
$R=\frac{c}{\frac{2\sqrt{3}}{2}}=\frac{c}{\sqrt{3}}$

Question26

If the area of a $\triangle ABC$ is $4\sqrt{5}$ sq units. Length of the side CA is 6 units and $\tan\frac{B}{2}=\frac{\sqrt{5}}{4}$, then its smallest side is of length

Options:

A.

5 units

B.

4 units

C.

3 units

D.

6 units

Answer: C

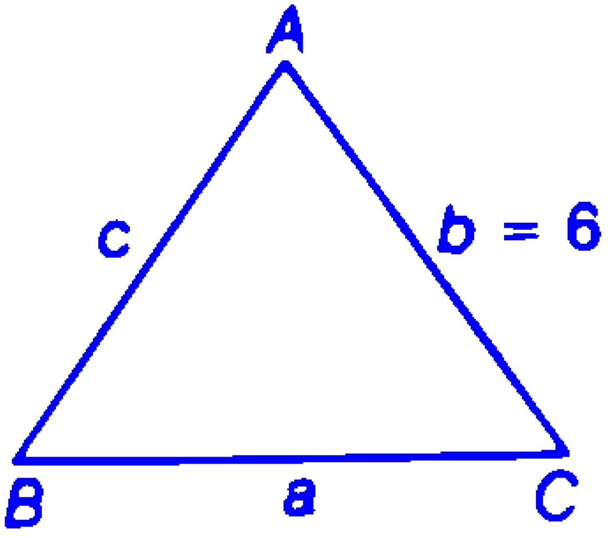
Solution:

We have, $\tan\frac{B}{2}=\frac{\sqrt{5}}{4}$ and $b=6$

Area of $\triangle ABC=\triangle=4\sqrt{5}$

$\cos B=\frac{1-\tan^2\frac{B}{2}}{1+\tan^2\frac{B}{2}}=\frac{11}{21}$

$$\Rightarrow \quad \sin B=\frac{\sqrt{320}}{21}=\frac{8\sqrt{5}}{21}$$
$$\because \quad \Delta=\frac{1}{2}ac\sin B=4\sqrt{5}$$
$$=\frac{1}{2}ac\frac{8\sqrt{5}}{21}=4\sqrt{5}$$



$\because \quad ac = 21 \qquad \ldots(i)$

$\because \quad b^2 = c^2 + a^2 - 2ac\cos B$

$\Rightarrow \quad 36 = c^2 + a^2 - 2 \times 21 \times \frac{11}{21}$

$\Rightarrow \quad 36 = c^2 + a^2 - 22$

$\Rightarrow \quad c^2 + a^2 = 58$

$\because \quad (c+a)^2 = c^2 + a^2 + 2ac$

$= 58 + 42 = 100$

$c + a = 10 \qquad \ldots(ii)$

$\Rightarrow \quad c = 7$ and $a = 3$

$\because$ Smallest sides is 3 units.

Question27

In a $\triangle ABC$ if $r_1 = 2r_2 = 3r_3$, then $a : b$ is

AP EAPCET 2025 - 21st May Morning Shift

Options:

A.

3 : 5

B.

5 : 3

C.

$4:5$

D.

$5:4$

Answer: D

Solution:

We have, $r_1 = 2r_2 = 3r_3$

$$\Rightarrow \quad \frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c} = \frac{1}{k} \quad \text{(say)}$$

$$\because s-a = k$$
$$s-b = 2k$$
$$s-c = 3k$$

$$3s-(a+b+c) = 6k$$
$$s = 6k$$
$$a = s-k = 5k$$
$$b = s-2k = 4k$$
$$\because \quad a:b = 5:4$$

Question28

In $\triangle ABC, a^2 \sin 2B + b^2 \sin 2A$ is equal to

AP EAPCET 2024 - 23th May Morning Shift

Options:

A. $2ab \cos A$

B. $2ab \sin A$

C. 2ab sin C

D. $2ab \cos C$

Answer: C

Solution:

In $\triangle ABC$, to evaluate the expression $a^2 \sin 2B + b^2 \sin 2A$, let's proceed with the following transformations:

$$\begin{aligned}
& a^2 \sin 2B + b^2 \sin 2A \\
& = 2a^2 \sin B \cos B + 2b^2 \sin A \cos A \\
& = 2a(b \sin A) \cos B + 2b^2 \sin A \cos A \\
& = 2b \sin A(a \cos B + b \cos A).
\end{aligned}$$

Now, applying the sine rule and angle sum identity in $\triangle ABC$:

$a \cos B + b \cos A = c.$
Thus, the expression simplifies to:
$= 2b \sin A(c).$

By the sine rule, we know that:

$$c = a \cos B + b \cos A \quad \Rightarrow \quad a^2 \sin 2B + b^2 \sin 2A = 2ab \sin C.$$

Therefore, the expression simplifies to $2ab \sin C$.

Question29

In $\triangle ABC, \frac{r_2(r_1+r_3)}{\sqrt{r_1r_2+r_2r_3+r_3r_1}}$ is equal to

AP EAPCET 2024 - 23th May Morning Shift

Options:

A. a

B. b

C. c

D. s

Answer: B

Solution:

Given, In $\triangle ABC$,

$$\frac{r_2\left(r_1+r_3\right)}{\sqrt{r_1 r_2+r_2 r_3+r_3 r_1}}=\frac{r_1 r_2+r_2 r_3}{\sqrt{r_1 r_2+r_2 r_3+r_3 r_1}}$$

$$=\frac{\frac{\Delta}{(s-a)} \times \frac{\Delta}{(s-b)}+\frac{\Delta}{(s-b)} \times \frac{\Delta}{(s-c)}}{\sqrt{\frac{\Delta}{s-a} \times \frac{\Delta}{s-b}+\frac{\Delta}{s-b} \times \frac{\Delta}{s-c}+\frac{\Delta}{s-c} \times \frac{\Delta}{s-a}}}$$

$$=\frac{\frac{\Delta^2(s-c+s-a)}{(s-a)(s-b)(s-c)}}{\Delta \sqrt{\frac{(s-c)+(s-a)+(s-b)}{(s-a)(s-b)(s-c)}}}$$

$$=\frac{\Delta(2 s-a-c)}{(s-a)(s-b)(s-c)} \times \frac{\sqrt{(s-a)(s-b)(s-c)}}{\sqrt{3 c-a-b-c}}$$

$$=\frac{\Delta(b)}{\sqrt{(s-a)(s-b)(s-c)} \sqrt{s}}$$

$$=\frac{\Delta(b)}{\sqrt{s(s-a)(s-b)(s-c)}}=b$$

Question30

In $\triangle ABC,\left(r_2+r_3\right) \operatorname{cosec}^2 \frac{A}{2}$ is equal to

AP EAPCET 2024 - 23th May Morning Shift

Options:

A. $4 R$

B. $4 R \cot ^2 \frac{\mathrm{A}}{2}$

C. $4 R \tan ^2 \frac{A}{2}$

D. $R \tan ^2 \frac{A}{2}$

Answer: B

Solution:

We have, in $\triangle ABC$

$$(r_2+r_3)\operatorname{cosec}^2\frac{A}{2}$$

$$=\left(\begin{array}{l}4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}\\+4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}\end{array}\right)\operatorname{cosec}^2\frac{A}{2}$$

$$=4R\cos\frac{A}{2}\left(\sin\frac{B}{2}\cos\frac{C}{2}+\cos\frac{B}{2}\sin\frac{C}{2}\right)$$

$$=4R\cos\frac{A}{2}\sin\left(\frac{B}{2}+\frac{C}{2}\right)\operatorname{cosec}^2\frac{A}{2}$$

$$=4R\cos\frac{A}{2}\cdot\cos\frac{A}{2}\operatorname{cosec}^2\frac{A}{2}$$

$$=4R\cot^2\frac{A}{2}$$

Question31

In a $\triangle ABC$, if $a=13, b=14$ and $c=15$, then $r_1=$

AP EAPCET 2024 - 22th May Evening Shift

Options:

A. $\frac{23}{2}$

B. $\frac{21}{2}$

C. $\frac{25}{2}$

D. $\frac{26}{2}$

Answer: B

Solution:

Given that in a $\triangle ABC$

$a=13, b=14$ and $c=15$

we know that $r_1=\frac{\Delta}{s-a}$

where, $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

and $s=\frac{a+b+c}{2}$

So, $s=\frac{13+14+15}{2}=\frac{42}{2}=21$

$$
\begin{aligned}
Now, \Delta &= \sqrt{21 \times 8 \times 7 \times 6} \\
&= \sqrt{3 \times 7 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3} \\
&= 2 \times 2 \times 3 \times 7 = 84
\end{aligned}
$$

So, $\quad r_1 = \frac{84}{21-13} = \frac{84}{8} = \frac{21}{2}$

Question32

In $a \triangle ABC$ if $r : R : r_2 = 1 : 3 : 7$, then $\sin(A + C) + \sin B$ is equal to

AP EAPCET 2024 - 22th May Evening Shift

Options:

A. 0

B. $\sqrt{3}$

C. 1

D. 2

Answer: D

Solution:

Given that

$r : R : r_2 = 1 : 3 : 7$

we know that $A + B + C = 180^{\circ}$

$$
\begin{aligned}
& Now, \quad \sin(A + C) + \sin B \\
& = \sin\left(180^{\circ} - B\right) + \sin B \\
& = \sin B + \sin B = 2 \sin B
\end{aligned}
$$

$$
\begin{aligned}
& r = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} = \frac{\Delta}{s} \\
& R = \frac{a}{2 \sin A} = \frac{b}{2 \sin B} = \frac{C}{2 \sin C} = \frac{abc}{4\Delta}
\end{aligned}
$$

On simplifying above ratio, we get

$B=90^{\circ}$
$\therefore \sin(A+C)+\sin B=2\sin 90^{\circ}=2\times 1=?$

Question33

In $\triangle ABC, (r_1+r_2)\operatorname{cosec}^2\frac{C}{2}$ is equal to

AP EAPCET 2024 - 22th May Evening Shift

Options:

A. $2R\cot^2\frac{C}{2}$

B. $4R\tan^2\frac{C}{2}$

C. $4R\cot^2\frac{C}{2}$

D. $2R\tan^2\frac{C}{2}$

Answer: C

Solution:

Given, in a $\triangle ABC$,

$A+B+C=180^{\circ}$

Now, $(r_1+r_2)\operatorname{cosec}^2\frac{C}{2}$

$$=\left(4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}+4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}\right)\cdot\operatorname{cosec}^2\frac{C}{2}$$
$$=4R\cos\frac{C}{2}\left[\sin\frac{A}{2}\cos\frac{B}{2}+\cos\frac{A}{2}\sin\frac{B}{2}\right]cosec^2\frac{C}{2}$$

$$=4R\cos\frac{C}{2}\left[\sin\left(\frac{A}{2}+\frac{B}{2}\right)\right]\operatorname{cosec}^2\frac{C}{2}$$

$$= 4R\cos\frac{C}{2}\cdot\sin\frac{(A+B)}{2}\cdot\operatorname{cosec}^2\frac{C}{2}$$
$$= 4R\cos\frac{C}{2}\cdot\frac{1}{\sin^2\frac{C}{2}}\sin\left(\frac{A+B}{2}\right)$$
$$= 4R\cot^2\frac{C}{2}\cdot\frac{\sin\left(\frac{A+B}{2}\right)}{\cos\frac{C}{2}}$$
$$= 4R\cot^2\frac{C}{2}\cdot\frac{\sin\left(90^\circ-\frac{C}{2}\right)}{\cos\frac{C}{2}}$$
$$= 4R\cot^2\frac{C}{2}\cdot\frac{\cos\frac{C}{2}}{\cos\frac{C}{2}} = 4R\cot^2\frac{C}{2}$$

Question34

In a $\triangle ABC$, if A, B and C are in arithmetic progression and $\cos A+\cos B+\cos C=\frac{1+\sqrt{2}+\sqrt{3}}{2\sqrt{2}}$, then $\tan A$:

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. $\sqrt{3}$

B. $2+\sqrt{3}$

C. 1

D. $2-\sqrt{3}$

Answer: B

Solution:

Given $\triangle ABC, A, B, C$ in AP which means $B-C=A-B$ where $C>B>A$

$\Rightarrow \quad 2B=A+C$

Sum of angles of $\triangle ABC$

$A+B+C=180^\circ \qquad \ldots(i)$

$\because A, B, C$ in AP, we can assume that $c=a$,

$B=a+d, A=a+2d$

From Eq. (i),

$$
\begin{aligned}
a+2d+a+d+a &= 180^\circ \\
3a+3d &= 180 \\
a+d = \tfrac{180}{3} &\Rightarrow B=60^\circ
\end{aligned}
$$

Thus, $A+C=120^\circ$

Given $\cos A+\cos B+\cos C=\frac{1+\sqrt{2}+\sqrt{3}}{2\sqrt{2}}$ $\quad\quad\ldots(ii)$

Let $C=45^\circ$ (we need to check for angles)

Then, $B=60^\circ, A=75^\circ$

Using Eq. (ii),

$$
\cos 75^\circ+\cos 60^\circ+\cos 45^\circ=\frac{\sqrt{3}-1}{2\sqrt{2}}+\frac{1}{2}+\frac{1}{\sqrt{2}}
$$

$$
=\frac{2+\sqrt{2}+\sqrt{3}-1}{2\sqrt{2}}=\frac{1+\sqrt{2}+\sqrt{3}}{2\sqrt{2}}
$$

This matches the expression.

Therefore, $A=75^\circ$ and $\tan A=\tan 75^\circ$

$\tan A=2+\sqrt{3}$

Question35

In $\triangle ABC$, if $b+c:c+a:a+b=7:8:9$, then the smaller angle (in radians) of that triangle is

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. $\cos^{-1}\left(\frac{4}{5}\right)$

B. $\frac{\pi}{3}$

C. $\cos^{-1}\left(\frac{3}{5}\right)$

D. $\frac{\pi}{4}$

Answer: A

Solution:

In triangle $\triangle ABC$, given the ratio of the sums of the sides as $b+c:c+a:a+b=7:8:9$, we can express these sums using a common factor k as follows:

$b+c=7k$

$c+a=8k$

$a+b=9k$

By adding up these equations, we have:

$b+c+c+a+a+b=7k+8k+9k=24k$

Simplifying, we get:

$2(a+b+c)=24k \quad \Rightarrow \quad a+b+c=12k$

Using this, we can solve for each side:

From $a+7k=12k$, it follows that $a=5k$.

From $c+a=8k$, substitute $a=5k$ to find $c=3k$.

From $a+b=9k$, substitute $a=5k$ to find $b=4k$.

Thus, the side lengths are:

$a=5k$

$b=4k$

$c=3k$

Next, we apply the cosine rule to find the angle C:

$\cos C=\frac{b^2+a^2-c^2}{2ba}$

Substituting the values, we have:

$$\begin{aligned}\cos C&=\frac{(4k)^2+(5k)^2-(3k)^2}{2\times 4k\times 5k}\\&=\frac{16k^2+25k^2-9k^2}{40k^2}\\&=\frac{32k^2}{40k^2}=\frac{4}{5}\end{aligned}$$

Therefore, the smallest angle C in radians is:

$C=\cos^{-1}\left(\frac{4}{5}\right)$

Question36

In $\triangle ABC$, if $(a+c)^2=b^2+3ca$, then $\frac{a+c}{2R}=$

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. $\frac{\sqrt{3}}{2}$

B. $\sqrt{3}\cos\left(\frac{A-C}{2}\right)$

C. $\cos\left(\frac{A-C}{2}\right)$

D. $\sin\left(\frac{A-C}{2}\right)$

Answer: B

Solution:

In the given triangle $\triangle ABC$, we are provided with the equation:

$(a+c)^2=b^2+3ac$

Using the cosine rule for triangles:

$$a^2=b^2+c^2-2bc\cos A$$
$$b^2=a^2+c^2-2ac\cos B$$
$$c^2=a^2+b^2-2ab\cos C$$

Additionally, we know the sum of angles in a triangle is π, i.e., $A+B+C=\pi$.

Given the equation:

$(a+c)^2=b^2+3ac$

We rearrange this equation:

$$a^2+c^2+2ac=b^2+3ac$$
$$a^2+c^2-b^2=ac \qquad (i)$$

From the cosine rule:

$a^2+c^2-b^2=2ac\cos B \qquad (ii)$

Equating Equation (i) and Equation (ii), we get:

$$ac=2ac\cos B$$
$$1=2\cos B$$
$$\cos B=\frac{1}{2}$$

Thus, $\angle B=\frac{\pi}{3}$ or 60°.

Therefore, we have:

$\frac{a+c}{2R} = \sin B \Rightarrow \sin 60^\circ = \frac{\sqrt{3}}{2}$

Thus:

$\frac{a+c}{2R} = \sqrt{3} \cos\left(\frac{A-C}{2}\right)$

Question37

In $\triangle ABC$, if A, B and C are in arithmetic progression $\Delta = \frac{\sqrt{3}}{2}$ and $r_1 r_2 = r_2 r$, then $R =$

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. $\sqrt{3}$

B. 2

C. 1

D. $\sqrt{2}$

Answer: C

Solution:

Given that the angles $\angle A, \angle B, \angle C$ in $\triangle ABC$ are in arithmetic progression, we can express $\angle B$ as the average of $\angle A$ and $\angle C$, i.e., $B = \frac{A+C}{2}$.

We also know that the sum of angles in a triangle is 180°:

$A + B + C = 180^\circ$

Substituting $B = \frac{A+C}{2}$ into this equation gives:

$A + \frac{A+C}{2} + C = 180^\circ$

$2A + A + C + 2C = 180^\circ \times 2$

$\frac{A+C}{2} = 60^\circ = B$

Therefore, $B = 60^\circ$.

The area of $\triangle ABC$ is given as $\frac{\sqrt{3}}{2}$.

Regarding the radii of the excircles, denoted as r_1, r_2, r_3 for the sides opposite to vertices A, B, C respectively, the condition given is:

$r_1 r_2 = r_2 r$

For any triangle, the area can also be expressed as:

$\frac{abc}{4R}$

Thus, equating the given area:

$\frac{\sqrt{3}}{2} = \frac{abc}{4R}$

Solving for R:

$R = \frac{abc}{2\sqrt{3}}$

Since we have determined that $B = 60^{\circ}$, the circumradius R in such a triangle is given by:

$R = \frac{b}{\sqrt{3}}$

Calculating the circumradius directly for special triangles:

Hence, the circumradius R is 1.

Question38

If 7 and 8 are the length of two sides of a triangle and a' is the length of its smallest side. The angles of the triangle are in AP and ' a ' has two values a_1 and a_2 satisfying this condition. If $a_1 < a_2$, then $2a_1 + 3a_2 =$

AP EAPCET 2024 - 21th May Evening Shift

Options:

A. 15

B. 21

C. 24

D. 28

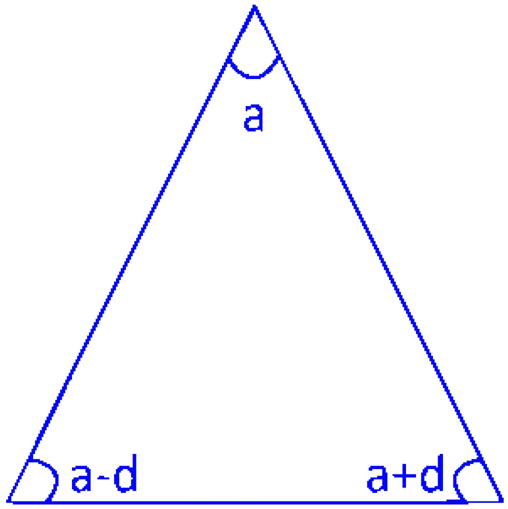
Answer: B

Solution:

We have, two larger side of a triangle are 8 and 7 and smaller side is a and angles are in AP

According to the given condition.

$$\Rightarrow \quad a+a-d+a+d=180^{\circ}$$
$$\Rightarrow \quad a=60^{\circ}$$



$$\text{Then,} \quad \cos 60^{\circ}=\frac{a^2+b^2-c^2}{2ac}$$
$$\Rightarrow \quad \frac{1}{2}=\frac{a^2+64-49}{2\times a\times 8}$$
$$\Rightarrow a^2-8a+15=0$$

On solving, we get

$$\Rightarrow \quad a=\frac{8\pm\sqrt{64-60}}{2}=\frac{8\pm 2}{2}$$
$$\Rightarrow \quad a_1=3a_2=5$$

Hence, the value satisfy the condition $a_1

Then, $2a_1+3a_2=2\times 3+3\times 5$

$$\Rightarrow \quad 2a_1+3a_2=21$$

Question39

In $\triangle ABC$, if $a=13, b=14$ and $\cos\frac{C}{2}=\frac{3}{\sqrt{13}}$, then $2r_1=$

AP EAPCET 2024 - 21th May Evening Shift

Options:

A. 2 S

B. Δ

C. S

D. 2Δ

Answer: C

Solution:

Given the triangle $\triangle ABC$ with sides $a = 13$, $b = 14$, and $\cos\left(\frac{C}{2}\right) = \frac{3}{\sqrt{13}}$.

Step 1: Utilize the Cosine Rule

First, we find $\cos C$ using:

$\cos C = 2\cos^2\left(\frac{C}{2}\right) - 1$

Substitute the given value:

$\cos C = 2\left(\frac{3}{\sqrt{13}}\right)^2 - 1 = 2 \times \frac{9}{13} - 1 = \frac{18-13}{13} = \frac{5}{13}$

Now, apply the cosine rule to find c:

$c^2 = a^2 + b^2 - 2ab\cos C$

Substitute the known values:

$c^2 = 13^2 + 14^2 - 2 \times 13 \times 14 \times \left(\frac{5}{13}\right)$

Calculate:

$c^2 = 365 - 140 = 225$

$c = 15$

Step 2: Calculate the Semi-perimeter

The semi-perimeter S is:

$S = \frac{a+b+c}{2} = \frac{13+14+15}{2} = \frac{42}{2} = 21$

Step 3: Find the Area using Heron's Formula

Calculate the area Δ:

$\Delta = \sqrt{S(S-a)(S-b)(S-c)} = \sqrt{21 \times (21-13) \times (21-14) \times (21-15)}$

$\Delta = \sqrt{21 \times 8 \times 7 \times 6} = 84$

Step 4: Calculate r_1

The radius r_1 of the excircle opposite A is:

$r_1 = \frac{\Delta}{S-a} = \frac{84}{21-13} = \frac{84}{8} = 10.5$

Multiplying by 2:

$2r_1 = 2 \times 10.5 = 21$

Conclusion

From the calculations, we find that $2r_1 = S$.

Question40

In $\triangle ABC$, if $(r_2 - r_1)(r_3 - r_1) = 2r_2r_3$, then $2(r + R) =$

AP EAPCET 2024 - 21th May Evening Shift

Options:

A. $a + b$

B. $c + a$

C. $2\sqrt{2}R\cos\left(\frac{C-A}{2}\right)$

D. $2\sqrt{2}R\cos\left(\frac{B-C}{2}\right)$

Answer: D

Solution:

Given, in a $\triangle ABC$

$$(r_2 - r_1)(r_3 - r_1) = 2r_2r_3$$
$$\Rightarrow \quad (r_1r_2 + r_2r_3 + r_1r_3) - r_1^2 = 0$$
$$\Rightarrow \quad \frac{r_1r_2r_3}{r} - r_1^2 = 0$$
$$\Rightarrow \quad r_1\left(\frac{r_2r_3}{r} - r_1\right) = 0$$
$$\Rightarrow \quad m_1 = r_2r_3 \quad (\because r_1 \neq 0)$$

$\Rightarrow \quad \frac{\Delta}{s} \cdot \frac{\Delta}{s-a} = \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c}$

$\Rightarrow \quad \frac{(s-b)(s-c)}{s(s-a)} = 1$

$\Rightarrow \quad \tan^2 \frac{A}{2} = \tan^2 45^\circ$

$\Rightarrow \quad A = 90^\circ$

$\therefore \quad \frac{B+C}{2} = 45^\circ$

Now, $2(r+R) = 2\left(4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} + R\right)$

$= 2R\left(4\left(\frac{1}{\sqrt{2}}\right)\sin\frac{B}{2}\sin\frac{C}{2} + 1\right)$

$= 2\sqrt{2}R\left(2\sin\frac{B}{2}\sin\frac{C}{2} + \frac{1}{\sqrt{2}}\right)$

$= 2\sqrt{2}R\left(2\sin\frac{B}{2}\sin\frac{C}{2} + \cos\left(\frac{B+C}{2}\right)\right)$

$= 2\sqrt{2}R\left(2\sin\frac{B}{2}\sin\frac{C}{2} + \cos\frac{B}{2}\cos\frac{C}{2} - \sin\frac{B}{2}\sin\frac{C}{2}\right)$

$= 2\sqrt{2}R\left(\cos\frac{B}{2}\cos\frac{C}{2} + \sin\frac{B}{2}\sin\frac{C}{2}\right) = 2\sqrt{2}R\cos\left(\frac{B-C}{2}\right)$

Question41

In a Δ if the angles are in the ratio $3 : 2 : 1$, then the ratio of its sides is

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. $1 : 2 : 3$

B. $2 : \sqrt{3} : 1$

C. $3 : \sqrt{2} : 1$

D. $1 : \sqrt{3} : 3$

Answer: B

Solution:

Given, angle ratio is $3 : 2 : 1$ We need to find sides Ratio.

$$
\begin{aligned}
&\text{Let } A = 3\theta, B = 2\theta \text{ and } C = \theta \\
\because\quad & A + B + C = \pi \quad \text{[angle sum property]} \\
& 3\theta + 2\theta + \theta = \pi \\
& 6\theta = \pi \\
& \theta = \frac{\pi}{6} \\
& A = 3 \times \frac{\pi}{6} = \frac{\pi}{2} \\
& B = 2 \times \frac{\pi}{6} = \frac{\pi}{3} \\
& C = \frac{\pi}{6} \\
& a : b : c = \sin\frac{\pi}{2} : \sin\frac{\pi}{3} : \sin\frac{\pi}{6} \\
\Rightarrow\quad & a : b : c = 1 : \frac{\sqrt{3}}{2} : \frac{1}{2} \\
& a : b : c = 2 : \sqrt{3} : 1
\end{aligned}
$$

Question42

In a $\triangle ABC$, if $BC = 5, CA = 6$ and $AB = 7$, then the length of the median drawn from B onto AC is

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. 5

B. $7\sqrt{5}$

C. $7\sqrt{2}$

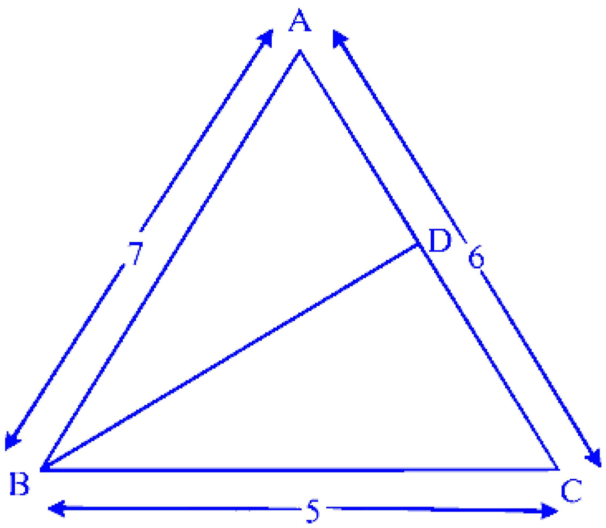
D. $2\sqrt{7}$

Answer: D

Solution:

In $\triangle ABC, BC = 5, CA = 6, AB = 7$.

Draw median from B onto $AC = 1$.



In $\triangle ABC$, point D is mid-point of AC.

$$AD = DC = \frac{1}{2}AC = 3$$

$(AB)^2 + (BC)^2 = 2BD^2 + 2DA^2$

[using Apollonius theorem]

$$\begin{aligned}(7)^2 + (5)^2 &= 2BD^2 + 2 \times (3)^2 \\ 49 + 25 &= 2BD^2 + 2 \times 9 \\ 74 &= 2BD^2 + 18 \\ 2BD^2 &= 74 - 18 = 56 \\ BD^2 &= 28 \\ BD^2 &= 7 \times 4 \Rightarrow BD = 2\sqrt{7}\end{aligned}$$

Hence, length of median drawn from B onto

$AC = 2\sqrt{7}$

Question43

In $\triangle ABC$, if $AB : BC : CA = 6 : 4 : 5$, then $R : r$ is equal to

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. 16 : 9

B. 16 : 7

C. 12 : 7

D. 12 : 9

Answer: B

Solution:

To find the ratio $R : r$ in $\triangle ABC$ with side ratios $AB : BC : CA = 6 : 4 : 5$, we use the following formula:

$\frac{R}{r} = \frac{abc}{4(s-a)(s-b)(s-c)}$

where s is the semi-perimeter, which can be calculated as:

$s = \frac{a+b+c}{2}$

Given the side ratios, let $a = 6x$, $b = 4x$, and $c = 5x$. Therefore, the semi-perimeter s is:

$s = \frac{6x+4x+5x}{2} = \frac{15x}{2}$

Substitute into the formula:

$\frac{R}{r} = \frac{6x \cdot 4x \cdot 5x}{4\left(\frac{15x}{2}-6x\right)\left(\frac{15x}{2}-4x\right)\left(\frac{15x}{2}-5x\right)}$

Calculate each term:

$s - a = \frac{15x}{2} - 6x = \frac{15x-12x}{2} = \frac{3x}{2}$

$s - b = \frac{15x}{2} - 4x = \frac{15x-8x}{2} = \frac{7x}{2}$

$s - c = \frac{15x}{2} - 5x = \frac{15x-10x}{2} = \frac{5x}{2}$

Substitute back in:

$\frac{R}{r} = \frac{6x \cdot 4x \cdot 5x}{4 \cdot \left(\frac{3x}{2}\right) \cdot \left(\frac{7x}{2}\right) \cdot \left(\frac{5x}{2}\right)}$

Simplify:

$\frac{R}{r} = \frac{120x^3}{4 \cdot \frac{3x}{2} \cdot \frac{7x}{2} \cdot \frac{5x}{2}} = \frac{120x^3}{4 \cdot \frac{105x^3}{8}}$

$= \frac{120x^3 \cdot 8}{4 \cdot 105x^3} = \frac{960}{420} = \frac{16}{7}$

Thus, $R : r$ is $\frac{16}{7}$.

Question44

If (α, β) is the orthocentre of the triangle with the vertices $(2,2),(5,1),(4,4)$, then $\alpha+\beta=$

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. 6

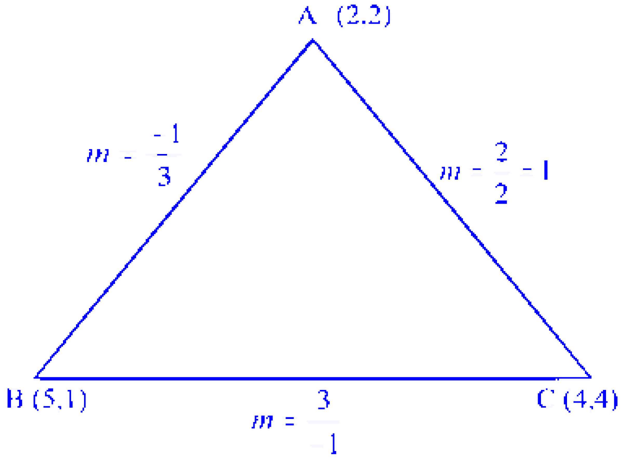
B. 5

C. $\frac{5}{2}$

D. $\frac{7}{2}$

Answer: A

Solution:



Altitude of BC,

$$y-2=\frac{1}{3}(x-2)$$
$$3y-6=x-2$$
$$x-3y+4=0$$

Altitude of AC, $y+1=-1(x-5)$

$$y-1=-x+5$$
$$x+y-6=0$$

From Eqs. (i) and (ii), we get

$-4y+10=0$

$$y=+\frac{5}{2}$$

$$\Rightarrow \quad \beta=\frac{5}{2} \Rightarrow x+\frac{5}{2}=6$$

$$\Rightarrow \quad 2x=12-5$$

$$x=\frac{7}{2}$$

$$\alpha=\frac{7}{2}$$

$$\Rightarrow \quad \alpha+\beta=\frac{7}{2}+\frac{5}{2}=\frac{12}{2}=6$$

$$\therefore \quad \alpha+\beta=6$$

Question45

In $\triangle ABC$, if $4r_1=5r_2=6r_3$, then $\sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2}=$

AP EAPCET 2024 - 20th May Evening Shift

Options:

A. $\frac{19}{22}$

B. $\frac{25}{33}$

C. $\frac{74}{99}$

D. $\frac{28}{33}$

Answer: B

Solution:

$\because \sin^2\frac{A}{2}=\frac{(s-b)(s-c)}{bc}$,

$\sin^2\frac{B}{2}=\frac{(s-c)(s-a)}{ac}$

and $\sin^2\frac{C}{2}=\frac{(s-a)(s-b)}{ab}$

$\Rightarrow \sin^2\frac{A}{2}+\sin^2\frac{B}{2}+\sin^2\frac{C}{2}$

$=\frac{(s-b)(s-c)}{bc}+\frac{(s-c)(s-a)}{ac}+\frac{(s-a)(s-b)}{ab} \quad \ldots$ (i)

Given, $4r_1=5r_2=6r_3$

$\Rightarrow r_1 : r_2 = 5 : 4$ and $r_2 : r_3 = 6 : 5$

$\Rightarrow r_1 : r_2 : r_3 = 30 : 24 : 20$

$\Rightarrow \frac{s}{s-a} : \frac{s}{s-b} : \frac{s}{s-c} = 30 : 24 : 20$

$\Rightarrow s - a : s - b : s - c = \frac{1}{30} : \frac{1}{24} : \frac{1}{20}$

$\therefore s - b + s - c = 2s - b - c = a$

$= \frac{1}{24} + \frac{1}{20} = \frac{11}{120}$

Similarly, $b = \frac{1}{30} + \frac{1}{20} = \frac{50}{600} = \frac{10}{120}$

and $\quad C = \frac{1}{30} + \frac{1}{24} = \frac{54}{720} = \frac{9}{120}$

$\therefore \quad a : b : c = 11 : 10 : 9$

$s = \frac{a+b+c}{2} = \frac{11+10+9}{2} = 15$

From Eq. (i), we get

$$\frac{\sin^2 A}{2} + \frac{\sin^2 B}{2} + \frac{\sin^2 C}{2}$$
$$= \frac{5 \times 6}{90} + \frac{6 \times 4}{99} + \frac{4 \times 5}{110}$$
$$\Rightarrow \quad \frac{30}{90} + \frac{24}{99} + \frac{20}{110} = \frac{25}{33}$$

Question46

In $\triangle ABC, r_1 \cot \frac{A}{2} + r_2 \cot \frac{B}{2} + m_3 \cot \frac{C}{2} =$

AP EAPCET 2024 - 20th May Evening Shift

Options:

A. 3Δ

B. $3S$

C. $\frac{s}{\Delta}$

D. Δ

Answer: A

Solution:

We know that

$r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}, r_3 = \frac{\Delta}{s-c}$

$\cot \frac{A}{2} = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}},$

$\cot \frac{B}{2} = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$

$\cot \frac{C}{2} = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}, r = \frac{\Delta}{s}$

Now, $\pi_1 \cot \frac{A}{2} + \pi_2 \cot \frac{B}{2} + \pi_3 \cot \frac{C}{2}$

$= \frac{\Delta}{s} \cdot \frac{\Delta}{s-a} \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} + \frac{\Delta}{s} \cdot \frac{\Delta}{s-b} \sqrt{\frac{s(s-b)}{(s-a)(s-c)}} + \frac{\Delta}{s} \cdot \frac{\Delta}{s-c} \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$$= \Delta^2 \sqrt{\frac{1}{s(s-a)(s-b)(s-c)}} + \Delta^2 \sqrt{\frac{1}{(s-a)(s-b)(s-c)s}} + \Delta^2 \sqrt{\frac{1}{(s-a)(s-b)(s-c)_1}}$$

$$= 3\Delta^2 \cdot \frac{1}{\sqrt{s(s-a)(s-b)(s-c)}} = 3\Delta$$

$$rr_1 \cot \frac{A}{2} + rr_2 \cot \frac{B}{2} + rr_3 \cot \frac{C}{2} = 3\Delta$$

Question47

In $\triangle ABC, bc - r_2 r_3 =$

AP EAPCET 2024 - 20th May Evening Shift

Options:

A.

rr_1

B.

rr_2

C. r_1

D. ar_1

Answer: A

Solution:

$$\begin{aligned}\triangle ABC, bc - r_2 r_3 &= bc - \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c} \\ &= bc - \frac{\Delta^2}{(s-b)(s-c)} \\ &= bc - \frac{s(s-a)(s-b)(s-c)}{(s-b)(s-c)} \\ &= bc - s(s-a) = (bc - s(s-a)) \\ &= bc\left[1 - \frac{s(s-a)}{bc}\right] \\ &= bc\left(1 - \cos^2 \frac{A}{2}\right) = bc \cdot \sin^2 \frac{A}{2} \\ &= bc \cdot \frac{(s-b)(s-c)}{bc} \\ &= \frac{(s-b)(s-c)(s-a) \cdot s}{s(s-a)} \\ &= \frac{\Delta^2}{s(s-a)} = \frac{\Delta}{s} \cdot \frac{\Delta}{s-a} \\ &= r \cdot r_1\end{aligned}$$

Question48

If $O(0,0,0)$, $A(3,0,0)$ and $B(0,4,0)$ form a triangle, then the incentre of $\triangle OAB$ is

AP EAPCET 2024 - 20th May Evening Shift

Options:

A. $(0,1,0)$

B. $(0,1,1)$

C. $(1,0,1)$

D. $(1,1,0)$

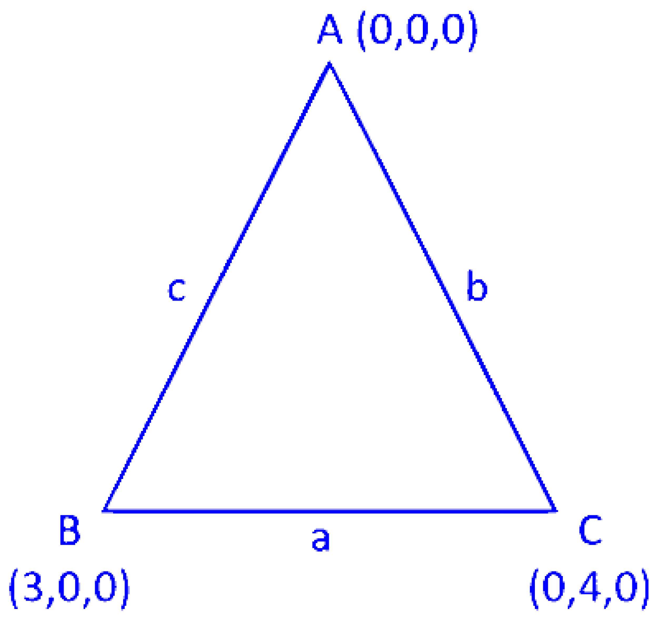
Answer: D

Solution:

The incentre of triangle formed by the points $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$ is

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}, \frac{az_1 + bz_2 + cz_3}{a+b+c}\right)$$

where a, b, c are the sides of the triangle. The vertices are $A(0,0,0)$, $B(3,0,0)$ and $C(0,4,0)$



Here, the sides are

$AB = 3, AC = 4$

and $BC = 5$

The incentre of the triangle is

$$\left(\frac{5(0)+4(3)+3(0)}{12}, \frac{5(0)+4(0)+3(4)}{12}, \frac{5(0)+4(0)+3(0)}{12}\right)$$
$$= \left(\frac{12}{12}, \frac{12}{12}, 0\right) = (1,1,0)$$

Question49

In $\triangle ABC$, if $r_1 = 4, r_2 = 8$ and $r_3 = 24$, then $a =$

AP EAPCET 2024 - 20th May Morning Shift

Options:

A. 0

B. $\frac{16}{\sqrt{5}}$

C. $16\sqrt{5}$

D. $\sqrt{5}$

Answer: B

Solution:

We have, $r_1 = 4, r_2 = 8$ and $r_3 = 24$

$$\begin{aligned}\frac{1}{r} &= \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{4} + \frac{1}{8} + \frac{1}{24} \\ &= \frac{6+3+1}{24} = \frac{10}{24} = \frac{5}{12} \Rightarrow r = \frac{12}{5}\end{aligned}$$

$$\Delta^2 = r_1 \cdot r_2 \cdot r_3 \cdot r = 4 \times 8 \times 24 \times \frac{12}{5}$$

So, $\Delta = \frac{96}{\sqrt{5}} \Rightarrow r_1 = \frac{\Delta}{s-a}$

$\Rightarrow s - a = \frac{\Delta}{r_1} = \frac{96}{\sqrt{5} \times 4} = \frac{24}{\sqrt{5}}$

Similarly, $s - b = \frac{\Delta}{r_2} = \frac{96}{\sqrt{5} \times 8} = \frac{12}{\sqrt{5}} \cdots$

$s - c = \frac{\Delta}{r_3} = \frac{96}{\sqrt{5} \times 24} = \frac{4}{\sqrt{5}} \cdots$

On adding Eqs. (i), (ii) and (iii), we get

$$s - a + s - b + s - c = \frac{24}{\sqrt{5}} + \frac{12}{\sqrt{5}} + \frac{4}{\sqrt{5}}$$

$$3s - (a + b + c) = \frac{40}{\sqrt{5}}$$

$$3s - 2s = \frac{40}{\sqrt{5}} \Rightarrow s = \frac{40}{\sqrt{5}}$$

Then, $a = s - \frac{24}{\sqrt{5}} = \frac{40}{\sqrt{5}} - \frac{24}{\sqrt{5}} = \frac{16}{\sqrt{5}}$

Question50

Match the items of List I with those of List II (here, Δ denotes the area of $\triangle ABC$)

List I		List II	
(A)	$\sum \cot A$	(i)	$(a+b+c)^2 \frac{1}{4\Delta}$
(B)	$\sum \cot \frac{A}{2}$	(ii)	$\left(a^2+b^2+c^2\right) \frac{1}{4\Delta}$
(C)	If $\tan A : \tan B : \tan C = 1 : 2 : 3$, then $\sin A : \sin B : \sin C =$	(iii)	$8 : 6 : 5$

List I		List II	
(D)	If $\cot\dfrac{A}{2}:\cot\dfrac{B}{2}:\cot\dfrac{C}{2}=3:7:9$ then $a:b:c=$	(iv)	$12:5:13$
		(v)	$\sqrt{5}:2\sqrt{2}:3$
		(vi)	4Δ

Then, the correct match is

AP EAPCET 2024 - 20th May Morning Shift

Options:

A. A-VI, B-I, C-II, D-III

B. A-II, B-I, C-V, D-III

C. A-II, B-VI, C-V, D-I

D. A-VI, B-II, C-I, D-IV

Answer: B

Solution:

Let a, b, c be the sides of $\triangle ABC$, tha

$$\cot A=\frac{b^2+c^2-a^2}{4\Delta}$$

$$\cot B=\frac{a^2+c^2-b^2}{4\Delta}, \cot C=\frac{a^2+b^2-c^2}{4\Delta}$$

So, $\Sigma\cot A=\cot A+\cot B+\cot C=\frac{1}{45}$

$[b^2+c^2-a^2+a^2+c^2-b^2+a^2+b^2-$

$$=\frac{a^2+b^2+c^2}{4\Delta}$$

(B) In $\triangle ABC$,

$$\cot\frac{A}{2}=\frac{s(s-a)}{\sqrt{s(s-a)(s-b)(s-c)}}=\frac{s(s-c)}{\Delta}$$

$$\cot\frac{B}{2}=\frac{s-b}{\Delta}$$

$$\cot\frac{c}{2}=\frac{s-c}{\Delta} \text{ where, } s=\frac{a+b+c}{2}$$

$$\Sigma\cot\frac{A}{2}=\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{c}{2}$$

$$\text{Thus, }=\frac{s(s-a)}{\Delta}+\frac{s(s-b)}{\Delta}+\frac{s(s-c)}{\Delta}$$

$$=\frac{s}{\Delta}[s-a+s-b+s-c]$$

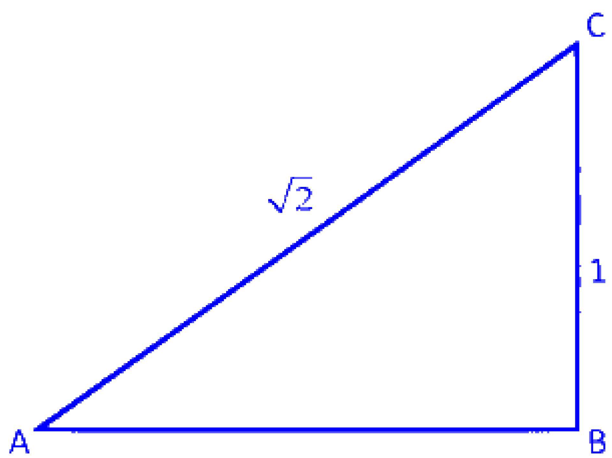
$$=\frac{s}{\Delta}[3s-(a+b+c)]$$

$$=\frac{s}{\Delta}(3s-2s)=\frac{s^2}{\Delta}=\frac{(a+b+c)^2}{4\Delta}$$

(C) We have, $\tan A:\tan B:\tan C=1:2:3$

So, $\tan A=k, \tan B=2K, \tan C=3K$

Now, $\tan C=\tan[\pi-(A+B)]=3K$



$$\Rightarrow \quad -\tan(A+B)=3K$$

$$\Rightarrow \quad \tan(A+B)=-3K$$

$$\Rightarrow \frac{\tan A+\tan B}{1-\tan A\tan B}=-3K$$

$$\Rightarrow \quad \frac{K+2K}{1-2K^2}=-3K$$

$$\Rightarrow \quad 1-2K^2=-1\Rightarrow 2K^2=2$$

$$\Rightarrow \quad K^2=1\Rightarrow K=1$$

$$\sin A=\frac{1}{\sqrt{2}}\Rightarrow\sin B=\frac{2}{\sqrt{5}}$$

$$\sin C=\frac{3}{\sqrt{10}}$$

$\sin A : \sin B : \sin C$

$$= \frac{1}{\sqrt{2}} : \frac{2}{\sqrt{5}} \frac{: 3}{\sqrt{10}} = \sqrt{5} : 2\sqrt{2} : 3$$

(D) We have,

$$\cot\left(\frac{A}{2}\right) : \cos\left(\frac{B}{2}\right) : \cot\left(\frac{C}{2}\right) = 3 : 5 : 7$$

$$\Rightarrow \frac{\cot A/2}{3} = \frac{\cot B/2}{7} = \frac{\cot C/2}{9} = k$$

$$\Rightarrow \frac{s(s-a)}{3\Delta} = \frac{s(s-b)}{7\Delta} = \frac{s(s-c)}{9\Delta}$$

$$\Rightarrow \frac{s-a}{3} = \frac{s-b}{7} = \frac{s-c}{9}$$

$$\Rightarrow \frac{s-b+s-c}{9+7} = \frac{s-a+s-c}{3+9}$$

$$\Rightarrow = \frac{s-a+s-b}{3+7}$$

$$\Rightarrow \frac{2s-b}{16} = \frac{a}{16} = \frac{b}{12} = \frac{c}{10} = K$$

$$\Rightarrow \quad a : b : c = 16 : 12 : 10 = 8 : 6 : 5$$

Question51

In a $\triangle ABC$, if $r_1 = 2r_2 = 3r_3$, then $\sin A : \sin B : \sin C =$

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. $5 : 4 : 2$

B. $3 : 4 : 2$

C. $6 : 3 : 2$

D. $5 : 4 : 3$

Answer: D

Solution:

$r_1 = 2r_2 = 3r_3$

$\Rightarrow \frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c}$

$\Rightarrow \frac{1}{s-a} = \frac{2}{s-b} = \frac{3}{s-c} = k \text{ (let)}$

$\Rightarrow s-a = \frac{1}{k}, s-b = \frac{2}{k} \text{ and } s-c = \frac{3}{k}, s = \frac{6}{k}$

$\Rightarrow a = s-(s-a) = \frac{5}{k}$

Similarly, $b = \frac{4}{k}$ and $c = \frac{3}{k}$

We know that $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \frac{\sin A}{5} = \frac{\sin B}{4} = \frac{\sin C}{3}$

$\Rightarrow \sin A : \sin B : \sin C = 5 : 4 : 3$

Question52

In $\triangle ABC$, if $B = 90^\circ$, then $2(r+R) =$

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. $a+b$

B. $b+c$

C. $a+c$

D. 0

Answer: C

Solution:

The circumradius R for a right-angled triangle is given by:

$\frac{b}{\sin B} = 2R$

Since $\sin 90^\circ = 1$, we have:

$b = 2R$

Therefore, $R = \frac{b}{2}$.

The inradius r is given by:

$r = (s-b)\tan\frac{B}{2} = (s-b)$

Since $\angle B = 90^\circ$, $\frac{B}{2} = 45^\circ$ and $\tan 45^\circ = 1$. Hence:

$r = s - b$

Now, we compute $2(r+R)$:

$2(r+R) = 2\left(s-b+\frac{b}{2}\right) = 2s-b$

Further simplifying, considering the semi-perimeter $s = \frac{a+b+c}{2}$:

$2s = a+b+c$

Therefore, substituting back:

$2s-b = a+b+c-b = a+c$

So, the value is $a+c$.

Question53

In a $\triangle ABC$, if $(a-b)(s-c) = (b-c)(s-a)$, then $r_1 + r_3 =$

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. $r_2 - r_3$

B. $2r_2$

C. $3r_2$

D. $3(r_1 + r_2)$

Answer: B

Solution:

In a triangle $\triangle ABC$, we have the relation:

$(a-b)(s-c) = (b-c)(s-a)$

Rearranging the terms, we obtain:

$(s-b)(s-c) + (s-b)(s-a) = 2(s-a)(s-c)$

Next, divide each term by $(s-a)(s-b)(s-c)$ and multiply by the area of the triangle Δ:

$\frac{\Delta}{s-a} + \frac{\Delta}{s-c} = \frac{2\Delta}{s-b}$

This simplifies to:

$r_1 + r_3 = 2r_2$

Question54

In $\triangle ABC$, $\cos A + \cos B + \cos C =$

AP EAPCET 2024 - 18th May Morning Shift

Options:

A. $\frac{1+\sqrt{2}}{R}$

B. $\frac{1}{R}$

C. $\frac{1+R}{R}$

D.

$$1 + \frac{r}{R}$$

Answer: D

Solution:

We have,

ABC is a triangle

$\cos A + \cos B + \cos C$

$$= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + 1 - 2\sin^2\frac{C}{2}$$

$$= 1 + 2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) - 2\sin^2\frac{C}{2} \qquad [\because A+B+C=\pi]$$

$$= 1 + 2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \sin\frac{C}{2}\right]$$

$$= 1 + 2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right]$$

$$= 1 + 2\sin\frac{C}{2}\left[2\sin\frac{A}{2}\sin\frac{B}{2}\right]$$

$$= 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$= 1 + 4\left[\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-a)(s-c)}{ac}}\right]$$

$$= \frac{1 + 4(s-a)(s-b)(s-c)}{abc}$$

$$= \frac{1 + 4s(s-a)(s-b)(s-c)}{sabc} [\therefore \text{ multiply and divide by } s]$$

$$= 1 + 4\frac{\Delta^2}{sabc}, \text{ where } \Delta \text{ is area of triangle}$$

$$= 1 + \left(\frac{\Delta}{s}\right)\left(\frac{4\Delta}{abc}\right) \quad \left[\because r = \frac{\Delta}{s} \text{ and } R = \frac{abc}{4\Delta}\right]$$

$$= 1 + \frac{r}{R}$$

Question55

In a $\triangle ABC$, if $a = 26, b = 30, \cos c = \frac{63}{65}$, then $c =$

AP EAPCET 2024 - 18th May Morning Shift

Options:

A. 2

B. 4

C. 6

D. 8

Answer: D

Solution:

In triangle $\triangle ABC$, given $a = 26$, $b = 30$, and $\cos C = \frac{63}{65}$, we need to find the length of side c.

We know the cosine rule for any triangle is given by:

$\cos C = \frac{a^2+b^2-c^2}{2ab}$

Substituting the known values:

$\frac{a^2+b^2-c^2}{2\times a\times b} = \frac{63}{65}$

This implied expression becomes:

$\frac{26^2+30^2-c^2}{2\times 26\times 30} = \frac{63}{65}$

Now, solve this step-by-step:

Calculate 26^2 and 30^2:

$26^2 = 676, \quad 30^2 = 900$

Substitute these into the equation:

$\frac{676+900-c^2}{1560} = \frac{63}{65}$

Simplify the numerator:

$676 + 900 = 1576$

Substitute back:

$\frac{1576-c^2}{1560} = \frac{63}{65}$

Cross-multiply to solve for c^2:

$65(1576 - c^2) = 63 \times 1560$

Compute 63×1560:

$63 \times 1560 = 98280$

Now, solve for c^2:

$65 \times (1576 - c^2) = 98280$

Simplify the expressions:

$102440 - 65c^2 = 98280$

Rearrange to find c^2:

$102440 - 98280 = 65c^2$

Solve:

$4160 = 65c^2$

Divide by 65 to isolate c^2:

$c^2 = \frac{4160}{65} = 64$

Take the square root:

$c = \sqrt{64} = 8$

Thus, the value of c is 8.

Question56

If H is orthocentre of $\triangle ABC$ and $AH = x; BH = y; CH = z$, then $\frac{abc}{xyz} =$

AP EAPCET 2024 - 18th May Morning Shift

Options:

A. 1

B. $\frac{a+b+c}{x+y+z}$

C. $\frac{a}{x} + \frac{b}{y} + \frac{c}{z}$

D. $\frac{ab+bc+ca}{xy+yz+zx}$

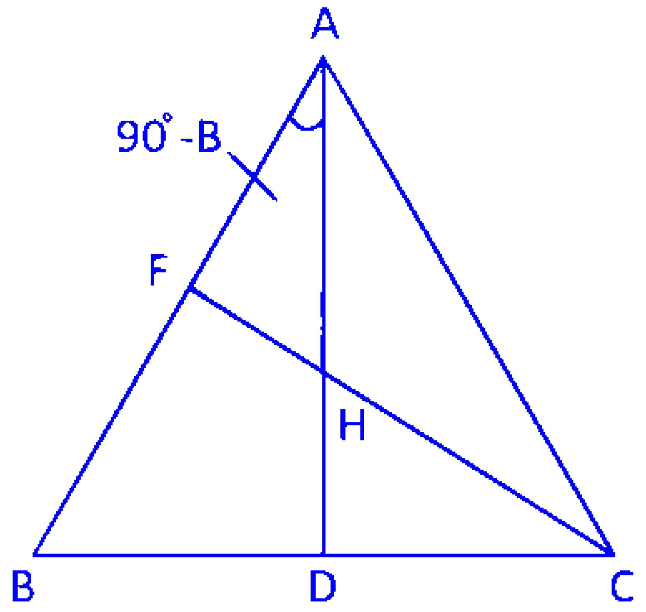
Answer: C

Solution:

In $\triangle AFC$

$\cos A = \frac{AF}{AC} = \frac{AF}{b}$

$\Rightarrow \quad AF = b \cos A$



In $\triangle AFH$

$\cos\left(90^\circ - 2B = \frac{AF}{AH} \Rightarrow \sin B = \frac{b\cos A}{x}\right.$

$\Rightarrow \quad x = \left(\frac{b}{\sin B}\right)\cos A$

Using sine rule, we get

$$x = \left(\frac{a}{\sin A}\right)\cos A$$
$$\Rightarrow \quad \frac{a}{x} = \tan A$$

Similarly, $\frac{b}{y} = \tan B$ and $\frac{c}{z} = \tan c$

$$\begin{aligned}\therefore \quad \frac{akc}{xyz} &= \tan A \tan B \tan C \\ &= \tan A + \tan B + \tan C \quad [\because A + B + C = \pi] \\ &= \frac{a}{x} + \frac{b}{y} + \frac{c}{z}\end{aligned}$$

Question57

In any $\triangle ABC$, $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} =$

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. $a^2 + b^2 + c^2$

B. $\frac{a^2+b^2+c^2}{2abc}$

C. $\frac{2abc}{a^2+b^2+c^2}$

D. $a + b + c$

Answer: B

Solution:

Given, in a $\triangle ABC$,

$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$

We know that,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$\text{Since, } \frac{b^2 + c^2 - a^2}{2abc} + \frac{a^2 + c^2 - b^2}{2abc} + \frac{a^2 + b^2 - c^2}{2ab}$$
$$= \frac{b^2 + c^2 - a^2 + a^2 + c^2 - b^2 + a^2 + b^2 - c^2}{2abc}$$
$$= \frac{a^2 + b^2 + c^2}{2abc}.$$

Question58

In a $\triangle ABC$, if $r_1 = 36, r_2 = 18$ and $r_3 = 12$, then $s =$

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. 6

B. 8

C. 16

D. 36

Answer: D

Solution:

Given,

$r_1 = 36, r_2 = 18$ and $r_3 = 12$

We know that, in a $\triangle ABC$

$$\begin{aligned} & \frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} \\ \Rightarrow & \frac{1}{r} = \frac{1}{36} + \frac{1}{18} + \frac{1}{12} \Rightarrow \frac{1}{r} = \frac{2+4+6}{72} \\ & r = 6 \end{aligned}$$

Now, we know that,

$$\begin{aligned} \Delta^2 &= r \cdot r_1 \cdot r_2 \cdot r_3 = 6 \times 36 \times 18 \times 12 \\ \Delta^2 &= 6^2 \times 6^2 \times 6^2 \\ \Delta &= 6 \times 6 \times 6 = 216 \end{aligned}$$

Again we know that,

$$\begin{aligned} & r = \frac{\Delta}{S} = \frac{216}{S} \\ \Rightarrow \quad & S = \frac{216}{6} = 36 \end{aligned}$$

Question59

In a $\triangle ABC, a = 6, b = 5$ and $c = 4$, then $\cos 2A =$

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. $-\frac{31}{32}$

B. $-\frac{15}{16}$

C. $\frac{31}{32}$

D. $\frac{15}{16}$

Answer: A

Solution:

In a $\triangle ABC, a=6$ units, $b=5$ units and $c=4$ units

As we know that,

$$\begin{aligned} \cos A &= \frac{b^2+c^2-a^2}{2bc} \\ \Rightarrow \cos A &= \frac{b^2+c^2-a^2}{2bc} = \frac{5^2+4^2-6^2}{2\times5\times4} = \frac{1}{8} \end{aligned}$$

As we know that,

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 = 2\times\left(\frac{1}{8}\right)^2 - 1 \\ &= \frac{1}{32} - 1 = \frac{-31}{32} \end{aligned}$$

Question60

In a $\triangle ABC$, $\left(\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}\right)^2 \leq$

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. $\frac{1}{27}$

B. $\frac{1}{18}$

C. $\frac{1}{9}$

D. $\frac{1}{3}$

Answer: A

Solution:

Within the context of triangle $\triangle ABC$, the following relationship holds:

We know that:

$\left(\tan \frac{A}{2} \cdot \tan \frac{B}{2} \cdot \tan \frac{C}{2}\right) \leq \frac{1}{3\sqrt{3}}$

Thus, squaring both sides of the inequality, we get:

$$\left(\tan \frac{A}{2} \cdot \tan \frac{B}{2} \cdot \tan \frac{C}{2}\right)^2 \leq \left(\frac{1}{3\sqrt{3}}\right)^2 = \frac{1}{27}$$

Question61

In a $\triangle ABC, 2(bc\cos A + ac\cos B + ab\cos C) =$

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. $a+b+c$

B. $2(a+b+c)$

C. $a^2+b^2+c^2$

D. $2\left(a^2+b^2+c^2\right)$

Answer: C

Solution:

Given that ABC be triangle.

Using cosine rule, $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos B = \frac{c^2+a^2-b^2}{2ca}, \cos C = \frac{a^2+b^2-c^2}{2ab}$

Now, $2(bc\cos A + ca\cos B + ab\cos C)$

$$= 2\left(bc\frac{b^2+c^2-a^2}{2bc} + ca\frac{c^2+a^2-b^2}{2ca} + ab\cdot\frac{a^2+b^2-c^2}{2ab}\right)$$

$$= 2\left(\frac{b^2+c^2-a^2}{2} + \frac{c^2+a^2-b^2}{2} + \frac{a^2+b^2-c^2}{2}\right)$$

$$= 2\left(\frac{b^2+c^2+a^2}{2}\right) = a^2 + b^2 + c^2$$

Question62

In a $\triangle ABC$, $\frac{a}{b} = 2 + \sqrt{3}$ and $\angle C = 60^\circ$. Then, the measure of $\angle A$ is

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. 95°

B. 65°

C. 105°

D. 115°

Answer: C

Solution:

Given that ABC is a triangle

So, $A + B + C = 180^\circ$

Given, $C = 60^\circ$

$\Rightarrow \quad A + B = 120^\circ \quad \ldots$ (i)

We have by sine formula

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\Rightarrow \frac{\sin A}{\sin B} = \frac{a}{b} = 2 + \sqrt{3} \text{ (given)}$$

Applying componendo and dividendo, we get

$$\frac{\sin A+\sin B}{\sin A-\sin B}=\frac{2-\sqrt{3}+1}{2+\sqrt{3}-1}=\frac{3+\sqrt{3}}{\sqrt{3}+1}$$

$$\Rightarrow \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}$$

$$=\frac{(3+\sqrt{3})(\sqrt{3}-1}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$\Rightarrow \frac{2\left(\frac{\sqrt{3}}{2}\right)}{2\left(\frac{1}{2}\right)}\cot\left(\frac{A-E}{2}\right)=\sqrt{3}$$

$$\Rightarrow \sqrt{3}\cot\left(\frac{A-B}{2}\right)=\sqrt{3}\Rightarrow\left(\frac{A-B}{2}\right)=45^{\circ}$$

$$\Rightarrow A-B=90 \quad \text{.... (ii)}$$

Solving Eqs. (i) and (ii), we get $\angle A=105^{\circ}$

Question63

If $a=2, b=3, c=4$ in a $\triangle ABC$, then $\cos C=$

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. $\frac{1}{4}$

B. $\frac{-1}{4}$

C. $\frac{1}{2}$

D. $\frac{-1}{2}$

Answer: B

Solution:

Given, $a=2, b=3, c=4$

Thus,

$$\cos C=\frac{a^2+b^2-c^2}{2ab}$$
$$=\frac{4+9-16}{2\times2\times3}=\frac{-3}{12}=\frac{-1}{4}$$

Question64

In a $\triangle ABC$ $(b+c)\cos A+(c+a)\cos B+(a+b)\cos C=$

AP EAPCET 2022 - 4th July Evening Shift

Options:

A. $2abc$

B. abc

C. $a+b+c$

D. $(a+b+c)/2abc$

Answer: C

Solution:

$$\begin{aligned}&(b+c)\cos A+(c+a)\cos B+(a+b)\cos C\\&=b\cos A+c\cos A+c\cos B+a\cos B+a\cos C+b\cos C\\&=(b\cos C+c\cos B)+(c\cos A+a\cos C)\\&\quad+(a\cos B+b\cos A)\quad\ldots\text{(i)}\end{aligned}$$

Using projection formula,

$$\begin{aligned}a&=(b\cos C+c\cos B)\\b&=(c\cos A+a\cos C)\\c&=(a\cos B+b\cos A)\end{aligned}$$

Substituting above values in Eq. (i), we get

$a+b+c$

Thus, $(b+c)\cos A+(c+a)\cos B+(a+b)\cos C=a+b+c$

Question65

Suppose $\triangle ABC$ is an isosceles triangle with $\angle C=90^{\circ}$, $A=(2,3)$ and $B=(4,5)$. Then, the centroid of the triangle is

Options:

A. $\left(\frac{13}{8}, \frac{8}{3}\right)$

B. $\left(\frac{11}{3}, \frac{10}{3}\right)$

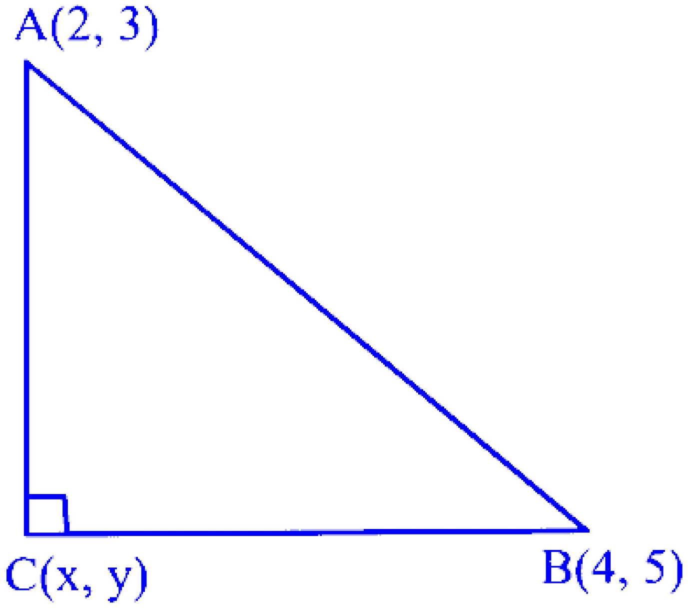
C. $\left(\frac{10}{3}, \frac{13}{3}\right)$

D. $\left(\frac{10}{3}, \frac{11}{3}\right)$

Answer: D

Solution:

Let the coordinates of C be (x, y).



Slope of $AC \times$ Slope of $BC = -1$

$$\left(\frac{y-3}{x-2}\right) \times \left(\frac{y-5}{x-4}\right) = -1$$

$\Rightarrow y^2 - 8y + 15 = -x^2 + 6x - 8$

$\Rightarrow x^2 + y^2 - 6x - 8y + 23 = 0 \quad$ ... (i)

$(AC)^2 = (BC)^2$[ABC is an isosceles right angled triangle]

$\Rightarrow (x-2)^2+(y-3)^2=(x-4)^2+(y-5)^2$
$\Rightarrow x^2+4-4x+y^2+9-6y$
$\quad = x^2+16-8x+y^2+25-10y$
$\Rightarrow 4x+4y=28 \Rightarrow x+y=7$
$\Rightarrow y=7-x \quad$ ... (ii)

Substituting the value of y from Eq. (ii) in the Eq. (i),

$\quad x^2+(7-x)^2-6x-8(7-x)+23=0$
$\Rightarrow x^2+49+x^2-14x-6x-56+8x+23=0$
$\Rightarrow 2x^2-12x+16=0 \Rightarrow x^2-6x+8=0$
$\Rightarrow x=4,2$

If $x=4$, then $y=3$ and If $x=2$, then $y=5$. If $(x,y)=(4,3)$, then

Centroid of triangle is $\left(\frac{2+4+4}{3}, \frac{3+3+5}{3}\right)=\left(\frac{10}{3}, \frac{11}{3}\right)$

If $(x,y)=(2,5)$, then centroid of $\triangle ABC$ is

$\left(\frac{2+2+4}{3}, \frac{5+3+5}{3}\right)=\left(\frac{8}{3}, \frac{13}{3}\right)$

Question66

In a $\triangle ABC$, if $a \neq b$, $\frac{a\cos A - b\cos B}{a\cos B - b\cos A}+\cos C=$

AP EAPCET 2022 - 4th July Morning Shift

Options:

A. 0

B. 1

C. 2

D. -1

Answer: A

Solution:

In $\triangle ABC, a \neq b$

$$\frac{a \cdot \cos A - b \cos B}{a \cos B - b \cos A} + \cos C$$

$$= \frac{a\left[\frac{b^2+c^2-a^2}{2bc}\right] - b\left[\frac{a^2+c^2-b^2}{2ac}\right]}{a\left[\frac{a^2+c^2-b^2}{2ac}\right] - b\left[\frac{b^2+c^2-a^2}{2bc}\right]} + \cos C$$

$$= \frac{a^2\left(b^2+c^2-a^2\right) - b^2\left(a^2+c^2-b^2\right)}{ab\left(a^2+c^2-b^2\right) - ab\left(b^2+c^2-a^2\right)} + \cos C$$

$$= \frac{a^2b^2 + a^2c^2 - a^4 - b^2a^2 - b^2c^2 + b^4}{a^3b + abc^2 - ab^3 - ab^3 - abc^2 + a^3b}$$

$$+ \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{a^2c^2 - a^4 - b^2c^2 + b^4}{2a^3b - 2ab^3} + \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{a^2c^2 - a^4 - b^2c^2 + b^4}{2ab\left(a^2-b^2\right)} + \frac{\left(a^2-b^2\right)\left(a^2+b^2-c^2\right)}{2ab\left(a^2-b^2\right)}$$

$$= \frac{a^2c^2 - a^4 - b^2c^2 + b^4}{2ab\left(a^2-b^2\right)}$$

$$+ \frac{a^4+a^2b^2-a^2c^2-b^2a^2-b^4+b^2c^2}{2ab(a^2-b^2)}$$

$$= \frac{\begin{array}{c} a^2c^2 - a^4 - b^2c^2 + b^4 + a^4 + a^2b^2 \\ -a^2c^2 - b^2a^2 - b^4 + b^2c^2 \end{array}}{2ab\left(a^2-b^2\right)}$$

$$= \frac{0}{2ab\left(a^2-b^2\right)} = 0$$

Question67

If in a $\triangle ABC, a = 2, b = 3$ and $c = 4$, then $\tan(A/2) =$

AP EAPCET 2022 - 4th July Morning Shift

Options:

A. $\sqrt{\frac{3}{15}}$

B. $\sqrt{\frac{4}{15}}$

C. $\sqrt{\frac{2}{15}}$

D. $\sqrt{\frac{1}{15}}$

Answer: D

Solution:

In a triangle $\triangle ABC$, the given sides are $a = 2$, $b = 3$, and $c = 4$.

We start by calculating the semi-perimeter (s) of the triangle:

$s = \frac{a+b+c}{2} = \frac{2+3+4}{2} = \frac{9}{2}$

Next, we use the formula for $\tan\left(\frac{A}{2}\right)$:

$\tan\left(\frac{A}{2}\right) = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

Substitute the values:

$\tan\left(\frac{A}{2}\right) = \sqrt{\frac{\left(\frac{9}{2}-3\right)\left(\frac{9}{2}-4\right)}{\frac{9}{2}\left(\frac{9}{2}-2\right)}} = \sqrt{\frac{\left(\frac{9}{2}-\frac{6}{2}\right)\left(\frac{9}{2}-\frac{8}{2}\right)}{\frac{9}{2}\left(\frac{9}{2}-\frac{4}{2}\right)}}$

Simplify the fractions:

$\tan\left(\frac{A}{2}\right) = \sqrt{\frac{(9-6)(9-8)}{9(9-4)}} = \sqrt{\frac{3\times1}{9\times5}}$

Finally, we have:

$\tan\left(\frac{A}{2}\right) = \sqrt{\frac{1}{15}}$

Question68

If the angles of a $\triangle ABC$ are in the ratio $1 : 2 : 3$, then the corresponding sides are in the ratio

AP EAPCET 2022 - 4th July Morning Shift

Options:

A. $\sqrt{3} : 2 : 1$

B. $1 : \sqrt{3} : 2$

C. $\sqrt{3} : 1 : 2$

D. $1:2:\sqrt{3}$

Answer: B

Solution:

The angles of $\triangle ABC$ are in ratio $1:2:3$.

Let $A=\theta, B=2\theta, C=3\theta$

$$\begin{aligned} \because\quad & A+B+C=180^{\circ} \\ \Rightarrow\quad & \theta+2\theta+3\theta=180^{\circ} \\ \Rightarrow\quad & 6\theta=180^{\circ} \\ \Rightarrow\quad & \theta=30^{\circ} \end{aligned}$$

$\therefore A=30^{\circ}, B=60^{\circ}$ and $C=90^{\circ}$

$$\because \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k$$

$$\begin{aligned} & a=k\sin A, b=k\sin B \text{ and } c=k\sin c \\ & a:b:c=k\sin 30^{\circ}:k\sin 60^{\circ}:k\sin 90^{\circ} \\ & \qquad=\frac{1}{2}:\frac{\sqrt{3}}{2}:1 \\ & \qquad=1:\sqrt{3}:2 \end{aligned}$$

Question69

In a $\triangle ABC, r_1 \cot \frac{A}{2}+r_2 \cot \frac{B}{2}+r_3 \cot \frac{C}{2}=$

AP EAPCET 2022 - 4th July Morning Shift

Options:

A. s

B. 2s

C. 3s

D. s/2

Answer: C

Solution:

We know, $r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b}$ and $r_3 = \frac{\Delta}{s-c}$

Also, $\cot\left(\frac{A}{2}\right) = \frac{s(s-a)}{\Delta}, \cot\left(\frac{B}{2}\right) = \frac{s(s-b)}{\Delta}$

and $\cot\left(\frac{C}{2}\right) = \frac{s(s-c)}{\Delta}$

$\therefore r_1 \cot\left(\frac{A}{2}\right) + r_2 \cot\left(\frac{B}{2}\right) + r_3 \cot\left(\frac{C}{2}\right)$

$= \frac{\Delta}{(s-a)} \cdot \frac{s(s-a)}{\Delta} + \frac{\Delta}{(s-b)} \cdot \frac{s(s-b)}{\Delta} + \frac{\Delta}{(s-c)} \cdot \frac{s(s-c)}{\Delta}$

$= s + s + s$
$= 3s$

Question70

What is the value of $(a-b)^2 \cos^2 \frac{c}{2} + (a+b)^2 \sin^2 \frac{c}{2}$ is equal to

AP EAPCET 2021 - 20th August Evening Shift

Options:

A. c^2

B. $a^2 + b^2$

C. $a^2 + b^2 + c^2$

D. $a^2 - b^2 + c^2$

Answer: A

Solution:

$$(a-b)^2\cos^2\frac{C}{2}+(a+b)^2\sin^2\frac{C}{2}$$
$$=\left(a^2+b^2\right)\left(\cos^2\frac{C}{2}+\sin^2\frac{C}{2}\right)+2ab\left(\sin^2\frac{C}{2}-\cos^2\frac{C}{2}\right)$$
$$=a^2+b^2+2ab\left(1-2\cos^2\frac{C}{2}\right)$$
$$=a^2+b^2+2ab(-\cos C)$$
$$=a^2+b^2-2ab\cos C$$
$$=a^2+b^2-2ab\left(\frac{a^2+b^2-c^2}{2ab}\right)$$
$$=a^2+b^2-a^2-b^2+c^2=c^2$$
$$\therefore (a-b)^2\cos^2\frac{C}{2}+(a+b)^2\sin^2\frac{C}{2}=c^2$$

Question71

In $\triangle ABC$, suppose the radius of the circle opposite to an angle A is denoted by r_1, similarly $r_2 \leftrightarrow$ angle $B, r_3 \leftrightarrow$ angle C. If $r_1=2, r_2=3$ and $r_3=6$, then what is (a, b, c) is equal to

AP EAPCET 2021 - 20th August Evening Shift

Options:

A. (3, 4, 5)

B. (3, 5, 4)

C. (5, 4, 3)

D. (5, 3, 4)

Answer: A

Solution:

$r_1=\frac{\Delta}{s-a}, r_2 \frac{\Delta}{s-b}$ and $r_3=\frac{\Delta}{s-c}$ ...... (i)

Also,

$$\Delta^2=r r_1 r_2 r_3 \quad .....\text{(ii)}$$
$$\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3} \quad .....\text{(iii)}$$

From Eq. (iii), we get

$$\frac{1}{r}=\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=\frac{3+2+1}{6}=1$$
$\therefore \quad r=1$

From Eq. (ii), we get $\Delta^2=(1)(2)(3)(6)=36$

$\Rightarrow \quad \Delta=6$

Also, $r=\frac{\Delta}{s}$

$$\Rightarrow \quad l=\frac{6}{s} \Rightarrow s=6$$
$$r_1=\frac{\Delta}{s-a}$$
$$\Rightarrow s-a=\frac{\Delta}{r_1} \Rightarrow a=s-\frac{\Delta}{r_1}$$
$$\Rightarrow \quad a=6-\frac{6}{2}=6-3$$
$$\Rightarrow \quad a=3 \Rightarrow r_2=\frac{\Delta}{s-b}$$
$$\Rightarrow \quad b=s-\frac{\Delta}{r_2}=6-\frac{6}{3} \Rightarrow b=4$$
$$r_3=\frac{\Delta}{s-c}$$
$$\Rightarrow \quad c=s-\frac{\Delta}{r_3}=6-\frac{6}{6}=6-1$$
$$\Rightarrow \quad c=5$$
$\therefore (a, b, c)=(3,4,5)$

Question72

If in $\triangle ABC, a \tan A+b \tan B=(a+b) . \tan \left(\frac{A+B}{2}\right)$, then which of the following holds?

AP EAPCET 2021 - 20th August Evening Shift

Options:

A. $A=B$

B. $A=2B$

C. $A=\frac{1}{2} B$

D. $A > B$

Answer: A

Solution:

$$a \tan A + b \tan B = (a+b) \tan \left(\frac{A+B}{2}\right)$$
$$\Rightarrow a\left(\tan A - \tan\left(\frac{A+B}{2}\right)\right) = b\left(\tan\left(\frac{A+B}{2}\right) - \tan B\right)$$
$$\Rightarrow a\left(\frac{\sin A}{\cos A} - \frac{\sin\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)}\right) = b\left(\frac{\sin\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} - \frac{\sin B}{\cos B}\right)$$
$$\Rightarrow a\left[\frac{\sin\left(A - \frac{(A+B)}{2}\right)}{\cos A \cos\left(\frac{A+B}{2}\right)}\right] = b\left[\frac{\sin\left(\frac{A+B}{2} - B\right)}{\cos B \cos\left(\frac{A+B}{2}\right)}\right]$$
$$a\left[\frac{\sin \frac{(A-B)}{2}}{\cos A}\right] = b\left[\frac{\sin \frac{(A-B)}{2}}{\cos B}\right]$$
$$\Rightarrow (k \sin A)\left[\frac{\sin\left(\frac{A-B}{2}\right)}{\cos A}\right] = (k \sin B)\left[\frac{\sin\left(\frac{A-B}{2}\right)}{\cos B}\right]$$
$$\left\{\begin{array}{l}\text{In } \triangle ABC \\ a = k \sin A \\ b = k \sin B\end{array}\right\}$$
$$\Rightarrow \quad \tan A = \tan B$$
$$\Rightarrow \quad \angle A = \angle B \Rightarrow A = B$$

Question73

In $\triangle ABC$, medians AD and BE are drawn. If $AD = 4, \angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of $\triangle ABC$ is

AP EAPCET 2021 - 20th August Morning Shift

Options:

A. $\frac{8}{3}$ sq units

B. $\frac{16}{3}$ sq units

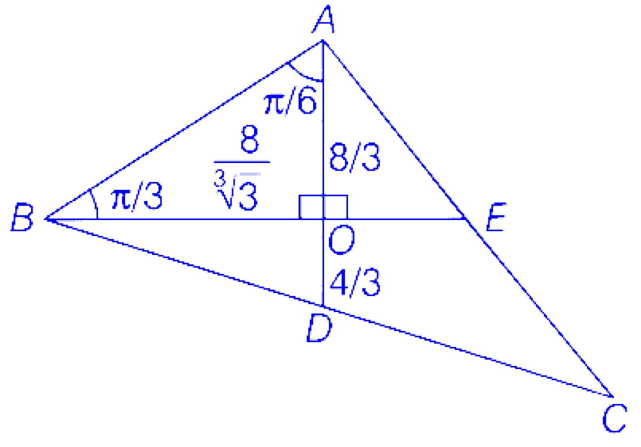
C.

$\frac{32}{3\sqrt{3}}$ sq. units

D. $\frac{64}{3}$ sq units

Answer: C

Solution:



Given : $AD = 4; \angle DAB = \frac{\pi}{6}; \angle ABE = \frac{\pi}{3}$

$AO = \frac{8}{3}$ and $OD = \frac{4}{3}$

$[\because O$ divides AD in $2:1]$

Area of $\triangle ABC = 2($Area of $\triangle ABE)$

$= 2\left[\frac{3}{2}(\text{ Area of } \triangle AOB)\right]$

$= 3 \times$ area of $\triangle AOB$

$= 3 \times \frac{1}{2} \times BO \times AO$

$= 3 \cdot \frac{1}{2} \cdot \frac{8}{3\sqrt{3}} \cdot \frac{8}{3}$

$\left\{\because \tan \frac{\pi}{6} = \frac{BO}{AO} \Rightarrow BO = \frac{1}{\sqrt{3}} \times \frac{8}{3}\right\}$

$= \frac{32}{3\sqrt{3}}$

Question74

In a $\triangle ABC, 2\Delta^2=\frac{a^2 b^2 c^2}{a^2+b^2+c^2}$, then the triangle is

AP EAPCET 2021 - 20th August Morning Shift

Options:

A. equilateral

B. isosceles

C. right angled

D. acute angled triangle

Answer: C

Solution:

$$2\Delta^2\left(a^2+b^2+c^2\right)=a^2 b^2 c^2$$
$$a^2+b^2+c^2=\left(\frac{abc}{\Delta}\right)^2 \cdot \frac{1}{2}=8R^2$$

[$\because abc/\Delta=4R$ where R-circumradius

$$\begin{aligned}
\because \quad & a=2R\sin A, b=2R\sin B, c=2R\sin C] \\
\Rightarrow & 4R^2\left(\sin^2 A+\sin^2 B+\sin^2 C\right)=8R^2 \\
\Rightarrow \quad & \sin^2 A+\sin^2 B+\sin^2 C=2 \\
\Rightarrow \quad & (\cos 2A+\cos 2B+\cos 2C)=-1 \\
\Rightarrow \quad & -1-4\cos A\cos B\cos C=-1
\end{aligned}$$
$$\cos A\cos B\cos C=0$$

So, any one among A, B and C has to be $\frac{\pi}{2}$.

So, right angled triangle.

Question75

In $\triangle ABC$, suppose the radius of the circle opposite to an angle A is denoted by r_1, similarly $r_2 \leftrightarrow$ angle $B, r_3 \leftrightarrow$ angle C. If $r_1 = 2, r_2 = 3, r_3 = 6$, what is the value of $r_1 + r_2 + r_3 - r =$ (R - radius of the circum circle).

AP EAPCET 2021 - 19th August Evening Shift

Options:

A. 4R

B. 3R

C. 2R

D. R

Answer: A

Solution:

$\because r = \frac{\Delta}{s}, r_1 = \frac{\Delta}{s-a}, r_2 = \frac{\Delta}{s-b},$

$r_3 = \frac{\Delta}{s-c}$ and $2s = a + b + c$

Then, $r_1 + r_2 + r_3 - r = \frac{\Delta}{s-a} + \frac{\Delta}{s-b} + \frac{\Delta}{s-c} - \frac{\Delta}{s}$

$$= \Delta\left(\frac{2s-(a+b)}{(s-a)(s-b)}\right) + \Delta\left(\frac{c}{s(s-c)}\right)$$
$$= \Delta\frac{(2s-2s+c)}{(s-a)(s-b)} + \Delta\left(\frac{c}{s(s-c)}\right)$$
$$= \Delta\left[\frac{cs(s-c)+c(s-a)(s-b)}{s(s-a)(s-b)(s-c)}\right]$$
$$[\because \Delta^2 = s(s-a)(s-b)(s-c)]$$
$$= \frac{\Delta c}{\Delta^2}(s(s-c)+(s-a)(s-b))$$
$$= \frac{c}{\Delta}\left(s^2 - sc + s^2 - (a+b)s + ab\right)$$
$$= \frac{c}{\Delta}\left(2s^2 - (a+b+c)s + ab\right)$$
$$= \frac{c}{\Delta}\left(2s^2 - 2s^2 + ab\right)[\because 2s = a+b+c]$$
$$= \frac{abc}{\Delta} = 4\left(\frac{abc}{4\Delta}\right) = 4R$$

Question76

In a ΔABC, if a = 3, b = 4 and $\sin A = \frac{3}{4}$, then $\angle CBA$ is equal to

AP EAPCET 2021 - 19th August Morning Shift

Options:

A. 60ϒ

B. 75ϒ

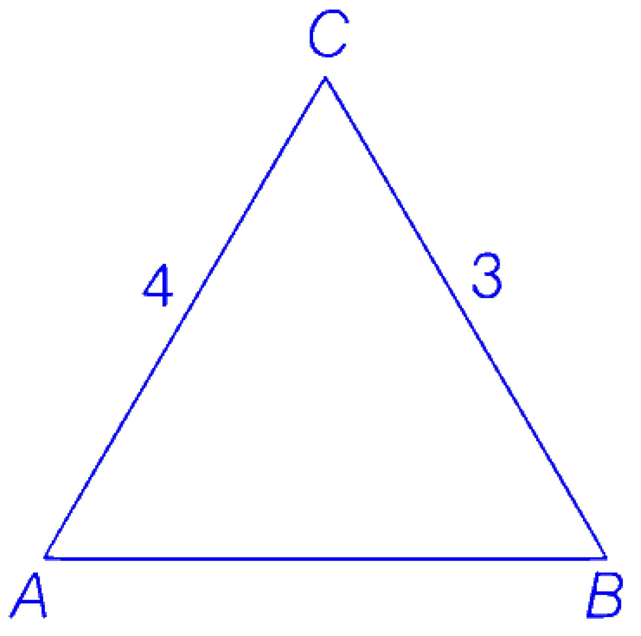
C. 90ϒ

D. 45ϒ

Answer: C

Solution:

In $\Delta ABC, a = 3, b = 4, \sin A = 3/4$



$\therefore \cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

But $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\Rightarrow \frac{\sqrt{7}}{4}=\frac{4^2+c^2-3^3}{2 . 4 . c}$

$\Rightarrow 2\sqrt{7}c = 7 + c^2$

$\Rightarrow c^2 - 2\sqrt{7}c + 7 = 0$

$\Rightarrow (c - \sqrt{7})^2 = 0$

$\therefore c = \sqrt{7}$

Now, $\cos B = \frac{c^2+a^2-b^2}{2ac}$

$\Rightarrow \cos B = \frac{7+9-16}{2 \cdot 3 \cdot \sqrt{7}} = 0 \Rightarrow B = 90\Upsilon$

$\therefore \angle CBA = 90\Upsilon$

Question77

In ΔABC, $A = 75\Upsilon$ and $B = 45\Upsilon$, then the value of $b + c\sqrt{2}$ is equal to

AP EAPCET 2021 - 19th August Morning Shift

Options:

A. a

B. 3a

C. 2a

D. 4a

Answer: C

Solution:

$A = 75Y, B = 45\Upsilon\$, then\$C = 180Y - (75Y + 45Y) = 60\Upsilon$

$\because \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{C}{\sin C} = 2R$ where R is circumradius

$\Rightarrow \quad a = 2R\sin A, b = 2R\sin B, c = 2R\sin C$

Then, $b + c\sqrt{2} = 2R(\sin B + \sqrt{2}\sin C)$

$= 2R(\sin 45Y + \sqrt{2}\sin 60Y)$

$= 2R\left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}}\right) = 2R\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)$

$= 2 \cdot 2R\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$

$= 2 \cdot (2R\sin 75Y)$

$= 2(2R\sin A)$

$= 2a$

Question78

In $\triangle ABC$, suppose the radius of the circle opposite to an $\angle A$ is denoted by r_1, similarly $r_2 \leftrightarrow \angle B$ and $r_3 \leftrightarrow \angle C$. If r is the radius of inscribed circle, then, what is the value of $\frac{ab-r_1r_2}{r_3}$ is equal to

AP EAPCET 2021 - 19th August Morning Shift

Options:

A. $r_1r_2r_3$

B. r

C. $r_1r_2\frac{r_3}{2}$

D. $\frac{r}{2}$

Answer: B

Solution:

In $\triangle ABC$

$a=2R\sin A=4R\sin\frac{A}{2}\cos\frac{A}{2}$
$b=2R\sin B=4R\sin\frac{B}{2}\cos\frac{B}{2}$
$r=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$
$r_1=4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
$r_2=4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$
$r_3=4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

$$ab-r_1r_2=16R^2\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{A}{2}$$

Now, $\cos\frac{B}{2}\left(1-\cos^2\frac{C}{2}\right)$

$$=16R^2\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{A}{2}\cos\frac{B}{2}\sin^2\frac{C}{2}$$

$\therefore\ \frac{ab-r_1r_2}{r_3}=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=r$

Question79

If D, E and F are respectively mid-points of AB, AC and BC in ΔABC, then BE + AF is equal to

AP EAPCET 2021 - 19th August Morning Shift

Options:

A. DC

B. $\frac{3}{2}$BF

C. $\frac{1}{2}$BF

D. $\frac{1}{2}$DC

Answer: A

Solution:

Let A, B, C is represented as $\mathbf{a}, \mathbf{b}, \mathbf{c}$ respectively.

Mid-points D, E, F is $\frac{\mathbf{a}+\mathbf{b}}{2}, \frac{\mathbf{a}+\mathbf{c}}{2}, \frac{\mathbf{b}+\mathbf{c}}{2}$ respectively.

$$\text{Then, } \mathbf{BE} = \frac{\mathbf{a}-2\mathbf{b}+\mathbf{c}}{2}, \mathbf{AF} = \frac{\mathbf{b}-2\mathbf{a}+\mathbf{c}}{2},$$
$$\mathbf{CD} = \frac{\mathbf{a}+\mathbf{b}-2\mathbf{c}}{2}$$
$$\mathbf{BE}+\mathbf{AF} = \frac{2\mathbf{c}-\mathbf{a}-\mathbf{b}}{2} = \frac{-(\mathbf{a}+\mathbf{b}-2\mathbf{c})}{2}$$
$$= -\mathbf{CD} = \mathbf{DC}$$
